LABOUR MARKET POWER, AUTOMATION AND WELFARE*

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Abstract

I develop and estimate a quantitative general equilibrium model of oligopsonistic labour markets with endogenous task-based automation, and provide conditions for a unique equilibrium and global identification of its parameters. Relative to an economy without wage-setting power, more productive firms employ too few workers and automate too many tasks. These distortions are larger when labour's comparative advantage rises less steeply across the tasks it still performs. The same schedule determines whether cheaper automation strengthens or weakens firms' wage-setting power. In line with the theory, French manufacturing firms with greater labour market power hold more automation capital. Across three specifications of labour's comparative advantage, removing labour market power raises welfare by 6.3–9.0% of consumption. A benchmark without automation understates this gain by 10–24%. Cheaper automation delivers almost all its gains to firm owners and raises the welfare cost of labour market power by 3–4% of its initial level.

Keywords: Labour market power, automation, task-based model, oligopsony, production function estimation.

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1 Introduction

This paper sits at the intersection of two burgeoning literatures. Extensive research on automation has shown that it can raise aggregate productivity and output but can also have adverse distributional consequences (Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018, 2019, 2022b; Eden and Gaggl, 2018; Hubmer and Restrepo, 2026; Humlum, 2021; Moll et al., 2022). Parallel work has shown that wage-setting power allows firms to pay workers below their marginal revenue product and distorts the allocation of workers across firms, with consequences for output and welfare (Berger et al., 2022; Card et al., 2018; Lamadon et al., 2022; Yeh et al., 2022).

I connect these literatures through a model in which automation and labour market power affect one another. Wage-setting power affects firms' incentives to automate, while automation can, in turn, strengthen or weaken firms' wage-setting power by changing their relative size within local labour markets.

This interaction helps rationalise why firms with greater labour market power automate more and implies that labour market power distorts not only employment but also firms' automation choices. Quantitatively, excluding automation understates the welfare gains from removing labour market power, while labour market power limits workers' gains from cheaper automation. Accounting for this interaction therefore matters for evaluating policies that address labour market power, such as minimum wages and collective bargaining, as well as redistributive policies such as progressive taxation.

To examine this interaction and quantify its welfare implications, I develop a quantitative general equilibrium model with oligopsonistic labour markets and endogenous automation choices. Firms compete for workers in local labour markets, where workers choose jobs on wages, amenities and idiosyncratic taste draws, which gives employers wage-setting power. Production consists of tasks

that firms can assign to labour or automation capital. Firms choose both whether to automate and, conditional on adoption, how many tasks to automate, with differences in productivity, amenities and installation costs generating variation in these choices. The equilibrium jointly determines the distributions of automation and labour market power across firms. I provide conditions under which the equilibrium exists and is unique.

The model delivers three analytical results. First, more productive firms employ more workers, pay higher wages, exercise greater wage-setting power and automate more tasks. Their larger relative size within the local labour market makes the labour supply they face less elastic, so attracting an additional worker requires a larger wage increase for existing workers. Together with their higher wages, this raises their marginal labour cost and encourages them to automate more tasks.

Second, wage-setting power distorts both employment and automation choices. Compared with the benchmark without labour markdowns, a small amount of wage-setting power moves workers away from more productive firms, which set the lower markdown and so reduce their posted wages by more. This reallocation raises marginal labour costs at more productive firms and lowers them at less productive ones, so the former automate more tasks and the latter fewer. More productive firms are therefore left with too few workers and too many automated tasks. These task distortions are larger when labour's comparative advantage schedule is relatively flat (i.e., labour's productivity relative to automation capital increases slowly across tasks), because a small change in relative costs then shifts more tasks between labour and automation capital.

Third, cheaper automation can strengthen or weaken the wage-setting power of more productive firms. Through the scale effect, it lowers firms' unit costs and encourages them to expand. Through the displacement effect, it shifts additional tasks from labour to automation capital. More productive firms benefit from a

larger scale effect because they already automate more. Whether their relative size within their local labour market increases depends on how much displacement they experience relative to less productive firms. Labour's comparative advantage schedule also governs this balance by determining the fraction of each firm's remaining labour tasks that becomes worth automating. If the stronger displacement effect at more productive firms outweighs their larger scale effect, their relative size and wage-setting power decline; when the larger scale effect dominates, both increase.

To assess the model's prediction that firms with greater labour market power automate more, I study the relationship between wage-setting power and automation among French manufacturing firms over 2009–2019. I measure automation using stocks of automation capital constructed from firms' import histories. These stocks capture the extensive margin, whether firms hold automation capital, and the intensive margin, how much they hold among adopters.

I measure firms' wage-setting power using two complementary approaches. First, I construct model-implied wage markdowns from firms' employment shares within local labour markets and the model's labour supply elasticities. I compute these employment shares using administrative workforce data, which I link to firms' import histories. Second, I estimate wage markdowns using the production-function approach. This requires additional information from firms' balance sheets and a production survey that reports prices and quantities separately. These data allow me to address biases in production-function estimation arising from unobserved productivity and variation in output and input prices.

Both approaches reveal the same pattern: within local labour markets and years, firms with greater wage-setting power are more likely to hold automation capital and, among adopters, hold more automation capital relative to lagged value added.

I show that the parameters are globally identified from the joint distribution of

wages, employment and revenue, and estimate the model by simulated method of moments using the 2019 cross-section of firms under three alternative comparative advantage schedules. These schedules differ in their *displacement elasticities*, that is, how readily firms replace their remaining labour tasks as automation becomes cheaper relative to labour, and in how these elasticities vary with the share of tasks already automated. This allows me to assess how the quantitative results depend on the displacement elasticities that govern the theoretical mechanisms.

To quantify how the interaction between automation and labour market power shapes welfare, I conduct two counterfactual experiments using the estimated model. First, I measure the welfare cost of labour market power with and without automation. I remove labour markdowns in two economies: one in which firms choose which tasks to automate, and another in which labour performs every task.

With endogenous automation, removing markdowns raises welfare by 6.3–9.0 per cent of 2019 consumption. The welfare gain from removing labour market power is larger with endogenous automation: abstracting from automation understates this gain by 10–24 per cent across the three estimated economies. In both economies, removing markdowns reallocates workers towards firms with greater wage-setting power and transfers a similar amount of income from owners to workers. The gain is larger with endogenous automation because more productive firms can perform part of their tasks with automation capital, which concentrates production in those firms without concentrating employment.[1] Employment then stays closer to the allocation workers prefer, which raises the value to workers of the income transferred to them.

Second, I examine how labour market power affects the magnitude of the gains from cheaper automation and their distribution between workers and firm owners. In each estimated economy, I lower the price of automation capital so

[1] In the model, workers choose jobs based on both wages and idiosyncratic preferences for workplaces. Concentrating employment at more productive firms draws some workers away from workplaces they prefer, lowering their welfare for a given aggregate wage bill.

that aggregate automation capital in the oligopsonistic economy grows in line with the projected growth in the number of industrial robots in France over 2019–2029. I apply the same price reduction to the corresponding economy without labour markdowns.

In the oligopsonistic economy, cheaper automation raises social welfare by 15–29 per cent of 2019 consumption, with almost all gains accruing to firm owners. Lower unit costs encourage firms to expand, but displacement limits the increase in labour demand and wages, allowing much of the benefit to accrue as profit. Labour market power further limits workers' gains, because firms with wage-setting power pay workers only a fraction of their marginal revenue product, so less of the gain from cheaper automation reaches them as wages. Cheaper automation raises social welfare more without labour markdowns than with them, increasing the welfare cost of labour market power by 3–4 per cent.

Related Literature. This paper contributes to five strands of the literature. First, it brings endogenous task automation into quantitative models of labour market power. Competitive task-based models explain how automation changes labour demand and its implications for the labour share (Acemoglu and Restrepo, 2018, 2019, 2020, 2022b; Hubmer and Restrepo, 2026), while models of imperfect labour market competition measure wage-setting power and show how it distorts the allocation of workers across firms, with consequences for welfare (Amodio et al., 2025; Armangué-Jubert et al., 2025; Azar et al., 2022; Azkarate-Askasua and Zerecero, 2025; Berger et al., 2022; Card et al., 2018; Deb et al., 2022, 2024; Gutiérrez, 2023; Jarosch et al., 2024; Lamadon et al., 2022).

My framework encompasses both competitive labour markets with endogenous automation and oligopsonistic labour markets without automation as special cases. I show that labour market power also distorts task allocation between labour and capital within firms, with the extent of misallocation depending on

labour's comparative advantage across tasks.[2]

Second, this paper contributes to the recent literature on automation and labour market power.[3] Azar et al. (2024) show that monopsonistic firms face higher marginal labour costs than wage-taking firms and therefore have stronger incentives to automate. Rubens (2025) identifies an investment holdup mechanism through which employer bargaining power encourages technology adoption.[4] Kikuchi (2023) documents a positive relationship between labour market power and labour share among similarly sized establishments in Japan, which he rationalises through weaker automation incentives in a framework similar to mine.

Relative to these papers, I make four contributions. First, I characterise how automation can, in turn, strengthen or weaken firms' wage-setting power, depending on labour's comparative advantage across tasks.[5] Second, I establish sufficient conditions for a unique equilibrium and for global identification of the model's parameters. Third, I develop a solution method that computes the equilibrium for a large number of heterogeneous firms and local labour markets. Fourth, I quantify the consequences of ignoring the interaction between automation and labour market power when evaluating the welfare cost of employer power and the gains from cheaper automation.

[2] Acemoglu and Restrepo (2026) obtain a related distortion from exogenous task-specific wage wedges. In their baseline model, firms take wages inclusive of these wedges as given, which can induce inefficient automation of high-rent tasks. Here, wage markdowns arise endogenously from firms' wage-setting power, and automation can in turn change that power by reallocating employment across firms.

[3] On the product-market side, Firooz et al. (2025) study the interaction between automation and heterogeneous markups using a CES technology over robots and labour. In their framework, product-market power distorts the allocation of production across firms but does not directly distort adopters' input mix at given input prices. Here, labour market power also distorts the allocation of tasks between labour and capital.

[4] Labour market power also affects the development of new technologies. Lehr (2026) shows that monopsony over inventors reduces innovation and growth by depressing R&D employment and distorting its allocation across firms.

[5] Automation can also affect employer power through wage bargaining. Leduc and Liu (2024) show that the option to automate strengthens firms' bargaining position. My mechanism operates through employment reallocation: automation changes firms' relative size in their local labour markets and hence the elasticity of the labour supply they face.

Third, this paper contributes to the empirical literature on automation. Most of this work studies its consequences for employment and firm performance (Acemoglu and Restrepo, 2020, 2022a; Aghion et al., 2023; Bonfiglioli et al., 2024; Dauth et al., 2021; Koch et al., 2021; Stiebale et al., 2024). A nascent literature asks how automation relates to labour market power (Byambasuren, 2026). I contribute by measuring both automation and wage-setting power at the firm level in French manufacturing, using model-implied markdowns and production-function estimates as complementary measures of wage-setting power. Both measures reveal a positive relationship between labour market power and automation, at both the extensive margin—whether firms adopt—and the intensive margin—how much automation capital adopters hold.

Fourth, this paper contributes to the literature on the welfare consequences of automation. Existing work studies its distributional consequences, across occupations (Eden and Gaggl, 2018; Humlum, 2021; Jaimovich et al., 2021; vom Lehn, 2020) and between workers and the owners of capital (Moll et al., 2022). I show that automation also has efficiency consequences. Through its scale and displacement effects, cheaper automation changes firms' relative size within their local labour markets and hence their wage-setting power, which governs how workers are misallocated across firms.

Finally, this paper connects to work on minimum wages (Berger et al., 2025; Hurst et al., 2025) and collective bargaining (Azkarate-Askasua and Zerecero, 2025) as responses to labour market power. These policies can encourage automation when they raise firms' marginal labour costs, as Brynjolfsson et al. (2026) document for US minimum wage increases. Where higher labour costs lead firms to reduce employment, automation can amplify these losses and limit gains to workers by replacing some of the jobs that would otherwise pay higher wages. Consequently, ignoring this response can understate employment losses and overstate gains to workers. My framework captures this interaction by jointly determining firms'

automation choices and wage-setting power.

Outline. Section 2 sets out the model, and Section 3 characterises the equilibrium. Section 4 extends the model for the quantitative analysis. Section 5 describes the data, presents evidence on the key mechanism and estimates the model. Section 6 performs counterfactual experiments, and Section 7 concludes. The Online Appendix contains the proofs and the solution algorithm, and the Supplemental Material contains derivations and additional information on the data and the estimation.

2 Model

The economy has four types of agent: workers, firm owners, a representative final-good producer, and a representative automation-capital producer. There is a mass $\mathcal{L}$ of workers, each supplying one unit of labour inelastically. Each intermediate-good firm has a distinct owner, who supplies no labour and consumes her profit. The final good is the numéraire, and automation capital is produced from it.

Intermediate firms produce differentiated varieties with a task-based technology in which each task can be performed by automation capital or by labour. They compete monopolistically in the product market and oligopsonistically in local labour markets. Each of the J local labour markets has a finite number N_j of firms, which post wages and compete à la Bertrand–Nash; each market is small relative to the economy, so firms take the economy-wide wage index as given.

2.1 Workers

Worker ℓ obtains indirect utility $u_{\ell ij} = w_i \, z_{\ell i} \, v_{\ell j}$ from working at firm i in market j, where w_i is the posted wage, $z_{\ell i}$ is an idiosyncratic taste draw for the firm, and $v_{\ell j}$ one for the local market. Both are drawn independently from Fréchet distributions: $z_{\ell i}$ has shape parameter ς and level parameter T_i, which I refer to as

firm i's amenity, and $v_{\ell j}$ has shape parameter η and unit level. I assume $\varsigma > 1$, so that $z_{\ell i}$ has a finite mean, and $0 < \eta < \varsigma$, so that tastes vary more across markets than within them, making jobs within a market closer substitutes than jobs in different markets. Labour supply to firm i is

$$l_i^s(w_i) \;=\; s_{i|j}\; s_j\; \mathcal{L}, \qquad s_{i|j} = \frac{T_i\, w_i^{\varsigma}}{\Phi_j}, \qquad s_j = \frac{\Gamma^{\eta}\, \Phi_j^{\eta/\varsigma}}{\Phi}, \tag{1}$$

where $s_{i|j}$ is firm i's within-market employment share and s_j is market j's share of total employment. $\Phi_j \equiv \sum_{i' \in j} T_{i'} w_{i'}^{\varsigma}$ and $\Phi \equiv \sum_{j'} \Gamma^{\eta} \Phi_{j'}^{\eta/\varsigma}$ are the market and economy wage indices, and $\Gamma \equiv \Gamma(1 - 1/\varsigma)$ is the gamma function evaluated at $1 - 1/\varsigma$. Supplemental Material S1.1 derives these expressions.

2.2 Producers

2.2.1 Final-good producer

A competitive representative producer combines the intermediate varieties $i \in \mathcal{I}$ into the final good with the CES technology $Y = \left(\sum_{i \in \mathcal{I}} y_i^{(\rho-1)/\rho}\right)^{\rho/(\rho-1)}$, where $\rho > 1$ is the elasticity of substitution. Each variety faces the inverse demand $p_i = (Y/y_i)^{1/\rho}$, so a firm's revenue is a power function of its own output,

$$r_i \;=\; p_i\, y_i \;=\; Y^{1/\rho}\, y_i^{1/\mu}, \qquad \mu \equiv \frac{\rho}{\rho - 1} \;>\; 1, \tag{2}$$

with Y taken as given by each firm and μ the demand-side markup.

2.2.2 Automation-capital producer

Following Guerreiro et al. (2022), a competitive producer makes automation capital from the final good: one unit requires c_k units of the final good. Its price therefore equals marginal cost, $p_k = c_k$, and the producer earns zero profit.

2.2.3 Intermediate-good producers

Technology. Each firm produces its variety from a measure M of tasks, indexed by $\tau \in [0, M]$. Output is a Cobb–Douglas aggregate of the tasks, with returns to scale $\kappa_f \in (0, 1)$ and Hicks-neutral productivity ϕ_{ih}. Within each task, automation capital and labour are perfect substitutes:

$$y_i = \phi_{ih} \left[\exp\left(\frac{1}{M} \int_0^M \log\big(M\, y_i(\tau)\big)\, d\tau \right) \right]^{\kappa_f}, \qquad y_i(\tau) = \phi_k(\tau)\, k_i(\tau) + \phi_l(\tau)\, l_i(\tau), \tag{3}$$

where $k_i(\tau)$ and $l_i(\tau)$ are the capital and labour allocated to task τ and $\phi_k(\cdot)$, $\phi_l(\cdot)$ are task-level factor productivities.

Assumption 1. *The task productivities ϕ_k, ϕ_l are strictly positive on $(0, M)$, and labour's comparative advantage, $R(\tau) \equiv \phi_l(\tau)/\phi_k(\tau)$, is continuously differentiable with $R'(\tau) > 0$ on $(0, M)$, with $R(\tau) \to 0$ as $\tau \to 0$ and $R(\tau) \to \infty$ as $\tau \to M$.*

Online Appendix A.1 shows that, under Assumption 1, at any factor prices almost every task is served by a single factor, and there is a unique $x_i \in (0, M)$ such that tasks below x_i use capital only and tasks above it use labour only. The task-level allocation can therefore be written

$$y_i(\tau) = \begin{cases} \phi_k(\tau)\, k_i(\tau), & \tau \le x_i, \\ \phi_l(\tau)\, l_i(\tau), & \tau > x_i, \end{cases} \tag{4}$$

where the firm chooses the automation threshold x_i, and $\tilde{x}_i \equiv x_i/M \in (0, 1)$ is the automated share of tasks.

The firm's problem. Let $k_i \equiv \int_0^{x_i} k_i(\tau)\, d\tau$ and $l_i \equiv \int_{x_i}^M l_i(\tau)\, d\tau$ denote total capital and employment. The firm chooses its wage, automation threshold and task inputs

to maximise profit,

$$\max_{w_i,\, x_i,\, k_i(\cdot),\, l_i(\cdot)} \quad r_i - c_k k_i - w_i l_i, \qquad \text{subject to} \quad l_i = l_i^s(w_i), \tag{5}$$

where revenue follows from demand and the task technology. Let ξ_i denote the multiplier on the labour-supply constraint.

Wage setting and marginal labour cost. Taking rivals' wages and the economy-wide wage index Φ as given, but internalising its effect on the market index Φ_j, the firm faces the labour-supply elasticity

$$\varepsilon_i \;\equiv\; \frac{d \log l_i^s}{d \log w_i} \;=\; \varsigma\,(1 - s_{i|j}) \;+\; \eta\, s_{i|j}. \tag{6}$$

Since $\eta < \varsigma$, firms with larger within-market employment shares face less elastic supply. The first-order condition for the posted wage,

$$-l_i + \xi_i\, l_i^{s\prime}(w_i) = 0,$$

together with the labour-supply constraint gives $\xi_i = w_i/\varepsilon_i$. Marginal labour cost is therefore

$$\omega_i \;\equiv\; w_i + \xi_i \;=\; \frac{w_i}{\delta_i}, \qquad \delta_i \equiv \frac{\varepsilon_i}{1+\varepsilon_i} \;\in\; (0,1). \tag{7}$$

Marginal labour cost exceeds the posted wage because attracting an additional worker requires raising the wage of all existing workers.

Task allocation and total factor demand. For a given threshold, the task-level input conditions equate marginal revenue products to marginal factor costs:

$$\frac{\kappa_f\, r_i}{\mu M\, k_i(\tau)} = c_k \quad (\tau < x_i), \qquad \frac{\kappa_f\, r_i}{\mu M\, l_i(\tau)} = \omega_i \quad (\tau > x_i). \tag{8}$$

These conditions imply uniform input quantities within each factor's task range,

$$k_i(\tau) = \frac{k_i}{x_i}, \qquad l_i(\tau) = \frac{l_i}{M - x_i}. \tag{9}$$

Multiplying each condition in (8) by its task input and integrating over the corresponding range gives

$$c_k\, k_i = \alpha_i\, r_i, \qquad \omega_i\, l_i = \left(\frac{\kappa_f}{\mu} - \alpha_i\right) r_i, \qquad \alpha_i \equiv \frac{\kappa_f}{\mu}\, \tilde{x}_i, \tag{10}$$

so that, by (7), the labour share of revenue is

$$\frac{w_i l_i}{r_i} = \delta_i\, \frac{\kappa_f}{\mu}\, (1 - \tilde{x}_i).$$

Capital expenditure is proportional to the share of tasks automated, while the labour share depends on both the share of tasks performed by labour and the wage markdown. Holding the markdown fixed, automating more tasks lowers labour's share of revenue. For a given task allocation, greater wage-setting power (a lower δ_i) also lowers this share by reducing workers' pay relative to their marginal revenue product.

Production and wage markdowns. Substituting the uniform allocations into the task technology gives production and revenue as functions of total inputs:

$$y_i = \tilde{\phi}_i(\tilde{x}_i)^{\kappa_f}\, k_i^{\mu\alpha_i}\, l_i^{\kappa_f - \mu\alpha_i}, \qquad r_i = Y^{1/\rho}\, \tilde{\phi}_i(\tilde{x}_i)^{\kappa_f/\mu}\, k_i^{\alpha_i}\, l_i^{\kappa_f/\mu - \alpha_i}, \tag{11}$$

where

$$\tilde{\phi}_i(\tilde{x}) \equiv \phi_{ih}^{1/\kappa_f} \left(\frac{\bar{\phi}_k(\tilde{x})}{\tilde{x}}\right)^{\tilde{x}} \left(\frac{\bar{\phi}_l(\tilde{x})}{1 - \tilde{x}}\right)^{1-\tilde{x}}.$$

Here

$$\bar{\phi}_k(\tilde{x}) \equiv \exp\Big\{\tilde{x}^{-1}\int_0^{\tilde{x}} \log\phi_k(Mu)\,du\Big\}, \qquad \bar{\phi}_l(\tilde{x}) \equiv \exp\Big\{(1-\tilde{x})^{-1}\int_{\tilde{x}}^{1} \log\phi_l(Mu)\,du\Big\},$$

are geometric mean productivities over each factor's task range.

Holding capital and the threshold fixed, the marginal revenue product of labour equals marginal labour cost at the optimum:

$$\mathrm{MRPL}_i = \Big(\frac{\kappa_f}{\mu} - \alpha_i\Big)\frac{r_i}{l_i} = \omega_i, \qquad w_i = \delta_i\,\mathrm{MRPL}_i. \tag{12}$$

The wage markdown therefore depends on the firm's within-market employment share:

$$\delta_i = \delta(s_{i|j}), \qquad \delta(s) \equiv \frac{\varsigma(1-s)+\eta s}{1+\varsigma(1-s)+\eta s}. \tag{13}$$

This function decreases from $\varsigma/(1+\varsigma)$ at $s=0$ to $\eta/(1+\eta)$ at $s=1$: firms with larger employment shares in their local labour market pay workers a smaller fraction of their marginal revenue product.

Optimal automation. At the marginal task, capital and labour have equal costs per efficiency unit:

$$\frac{c_k}{\phi_k(x_i)} = \frac{\omega_i}{\phi_l(x_i)} \quad\Longleftrightarrow\quad R(x_i) = \frac{\omega_i}{c_k}. \tag{14}$$

Under Assumption 1, R increases from zero to infinity, giving the unique interior automation rule

$$\tilde{x}_i = \frac{1}{M}\,R^{-1}\Big(\frac{\omega_i}{c_k}\Big) \;\in\; (0,1). \tag{15}$$

A higher marginal labour cost raises the automation threshold, so the firm assigns more tasks to capital and fewer to labour.

Displacement elasticity. The task schedule determines how far changes in relative factor costs move the automation threshold:

$$\frac{d\log(1-\tilde{x}_i)}{d\log(\omega_i/c_k)} = -\,h(\tilde{x}_i), \qquad h(\tilde{x}) \equiv \left[(1-\tilde{x})\,\frac{d\log R(M\tilde{x})}{d\tilde{x}}\right]^{-1}. \tag{16}$$

The displacement elasticity $h(\tilde{x}_i)$ measures the percentage reduction in the measure of tasks performed by labour following a one per cent increase in marginal labour cost relative to the capital price. At a given threshold, the elasticity is lower where labour's comparative advantage rises more steeply across tasks, because fewer tasks switch to capital following a given cost increase. As the automated share increases, the displacement elasticity is increasing if $h'(\tilde{x}) > 0$, constant if $h'(\tilde{x}) = 0$, and decreasing if $h'(\tilde{x}) < 0$ on all of $(0,1)$.

Profit. Substituting factor demands and the wage rule into profit gives

$$\tilde{\pi}_i = r_i - c_k k_i - w_i l_i = \Bigg[\underbrace{\left(1-\tfrac{\kappa_f}{\mu}\right)}_{\substack{\text{rent from the markup}\\ \text{and decreasing returns}}} + \underbrace{\left(1-\delta_i\right)\tfrac{\kappa_f}{\mu}\left(1-\tilde{x}_i\right)}_{\text{labour-market rent}} \Bigg] r_i. \tag{17}$$

The first term is the profit share when labour is paid its marginal revenue product. The second is the additional share arising from wage markdowns.

2.3 Aggregation and market clearing

Factor markets. Each worker supplies one unit of labour, so aggregate employment equals $\mathcal{L}$. Aggregate capital follows from firms' capital demands:

$$\sum_{i\in\mathcal{I}} l_i = \mathcal{L}, \qquad K \;\equiv\; \sum_{i\in\mathcal{I}} k_i \;=\; \frac{1}{c_k}\sum_{i\in\mathcal{I}} \alpha_i\, r_i. \tag{18}$$

Aggregate output. The competitive final-good producer earns zero profit, so $Y = \sum_i r_i$. Substituting factor demands into firm revenue gives

$$r_i = Y^{1/[\rho(1-\kappa_f/\mu)]}\,\Xi_i, \tag{19}$$

where

$$\Xi_i \equiv \left[\tilde{\phi}_i^{\kappa_f/\mu}\left(\frac{\alpha_i}{c_k}\right)^{\alpha_i}\left(\frac{\kappa_f/\mu-\alpha_i}{\omega_i}\right)^{\kappa_f/\mu-\alpha_i}\right]^{1/(1-\kappa_f/\mu)},$$

with α_i and $\tilde{\phi}_i$ evaluated at the firm's equilibrium automation threshold. Summing across firms and solving for aggregate output yields

$$Y = \left(\sum_{i\in\mathcal{I}}\Xi_i\right)^{\frac{\rho(1-\kappa_f/\mu)}{(\rho-1)(1-\kappa_f)}}. \tag{20}$$

Consumption. Let $\mathcal{B} \equiv \sum_i w_i l_i$ denote the aggregate wage bill and $\Pi \equiv \sum_i \tilde{\pi}_i$ aggregate profit. Workers consume their wages and owners their profits. Summing firms' budget identities therefore gives

$$C = \mathcal{B} + \Pi = Y - c_k K. \tag{21}$$

Consumption equals output net of the resources used to produce automation capital. Workers receive the share $\sigma \equiv \mathcal{B}/C$ of consumption.

Social welfare. Following Azkarate-Askasua and Zerecero (2025), I use a representative-household formulation that generates the same labour supply as the worker-choice problem in Section 2.1. Its maximised utility is $\log\mathcal{W}$, where $\mathcal{W} = \Phi^{1/\eta}$. This provides a welfare measure even when $\eta \leq 1$, for which mean utility in the worker-choice model is infinite.

In consumption-equivalent terms, workers' welfare is $\mathcal{B}\mathcal{N}/\bar{\mathcal{N}}$, where $\mathcal{B}$ is the aggregate wage bill, the employment index $\mathcal{N} \equiv \mathcal{W}/\mathcal{B}$ captures the welfare effect of employment allocation, and $\bar{\mathcal{N}}$ is its value at the reference allocation. Owners

have linear utility, so their consumption-equivalent welfare equals profits, $\mathcal{W}^f = \Pi$. Social welfare is the sum of the two groups' consumption equivalents:

$$\mathcal{S} \;=\; \mathcal{B}\,\frac{\mathcal{N}}{\bar{\mathcal{N}}} + \Pi \;=\; \sigma\, C\,\frac{\mathcal{N}}{\bar{\mathcal{N}}} \;+\; (1-\sigma)\, C. \tag{22}$$

Supplemental Material S1.1 provides additional details.

2.4 Equilibrium

Definition 1. Given firm draws (ϕ_{ih}, T_i), an equilibrium consists of wages w_i, employment l_i, automated shares $\tilde{x}_i$, capital k_i, output y_i and profit $\tilde{\pi}_i$ for each firm; market indices Φ_j and the aggregate index Φ; and aggregate output Y, such that (i) workers choose employers optimally, so $l_i = l_i^s(w_i)$ as in (1); (ii) each firm maximises profit (5), taking Y, Φ and rivals' wages as given, so its wage, automated share and inputs satisfy the wage equation (12), the automation rule (15) and the factor demands (10); and (iii) the goods market clears, $Y = \sum_i r_i$.

Proposition 1. *Under regularity conditions, the equilibrium exists and is unique.*

The proof is in Online Appendix A.2. It also covers the extensions in Section 4.

2.5 Special cases

Competitive labour markets. As workers' taste heterogeneity vanishes, $\varsigma, \eta \to \infty$, labour supply to each firm becomes perfectly elastic. All firms pay the same wage and have $\delta_i = 1$. They therefore face the same marginal labour cost and choose the same automated share, $\tilde{x}_i = \tilde{x} > 0$. Automation remains endogenous, but neither wage markdowns nor automated shares vary across firms.

Oligopsonistic labour markets without automation. If automation capital cannot perform any task, $\phi_k(\tau) = 0$ for all τ, labour performs every task and $\tilde{x}_i = 0$. The model then reduces to a labour-only model of oligopsonistic labour markets.

Wage markdowns depend on firms' employment shares within their local labour markets, $\delta_i = \delta(s_{i|j})$, and can therefore differ across firms.

3 Characterisation

This section works with the *analytical economy*: one local labour market with two firms, a high-productivity firm H and a low-productivity firm L, $\phi_{Hh} > \phi_{Lh}$, with a common amenity T. The single market employs the whole labour force.[6] Proofs are in Online Appendix A. In Supplemental Material S2, I extend the analysis to two types of labour market. Propositions 2 and 4 below carry over, and Proposition 3 holds under an additional condition.

3.1 Equilibrium ranking

Proposition 2. *The more productive firm:*

(i) is larger: $s_H > s_L$, $l_H > l_L$, *and* $r_H > r_L$;

(ii) pays higher wages and exercises more labour market power: $w_H > w_L$ *and* $\delta_H < \delta_L$, *hence* $\omega_H > \omega_L$;

(iii) automates more: $\tilde{x}_H > \tilde{x}_L$;

(iv) has the higher profit margin, $\tilde{\pi}_H / r_H > \tilde{\pi}_L / r_L$, *if and only if*

$$(1 - \tilde{x}_H)(1 - \delta_H) \;>\; (1 - \tilde{x}_L)(1 - \delta_L). \tag{23}$$

The proof is in Online Appendix A.3.

Proposition 2 shows that the more productive firm is larger, pays a higher wage and automates more. Its larger employment share makes its labour supply less elastic, so it sets a lower markdown, $\delta_H < \delta_L$. A lower markdown raises the

[6] Each firm still takes Φ as given when it sets its wage, so the labour supply elasticity it perceives is (6) and its markdown depends on η, even though the market employs every worker.

marginal labour cost above the posted wage, because hiring one more worker raises the wage of every worker the firm already employs. Together with its higher wage, $w_H > w_L$, this raises its marginal labour cost, $\omega_H > \omega_L$, and therefore its incentive to automate:

$$\tilde{x}_i \;=\; \frac{1}{M}\, R^{-1}\!\Big(\frac{w_i}{\delta_i\, c_k}\Big). \tag{24}$$

Part (iv) shows why the more productive firm need not earn a higher profit margin. In (17), the rent from the markup and decreasing returns, $1 - \kappa_f/\mu$, is common to both firms. The ranking therefore depends on the labour-market rent, $(1-\delta_i)(\kappa_f/\mu)(1-\tilde{x}_i)$. The productive firm has greater wage-setting power but automates more tasks, which reduces its scope to earn this rent. Its margin is higher only when its greater wage-setting power outweighs its smaller share of tasks performed by labour.[7]

3.2 From labour market power to automation

The distortions from wage-setting power are measured against the benchmark *equilibrium without labour markdowns*, in which firms face the same labour supply as in Section 2 but pay the marginal revenue product, $w_i = \text{MRPL}_i$ and $\delta_i = 1$. To move between this benchmark and the equilibrium of Definition 1, parametrise the wage rule by $\theta \in [0,1]$,

$$w_i \;=\; \big\{1 - \theta\,[1 - \delta(s_{i|j})]\big\}\,\omega_i, \tag{25}$$

so that $\theta = 0$ is the benchmark and $\theta = 1$ is the wage-posting rule (12).

Proposition 3. *Starting from the benchmark equilibrium without labour markdowns, a marginal introduction of wage-setting power, $d\theta > 0$ at $\theta = 0$,*

[7] The displacement elasticity (16) governs this trade-off. A high elasticity means that a given increase in the marginal labour cost displaces a larger fraction of the remaining labour tasks. If this displacement outweighs the productive firm's greater wage-setting power, its margin is lower and profit is less concentrated than revenue.

(i) moves workers from firm H to firm L: $ds_H/d\theta < 0$, *and the wage premium* w_H/w_L *falls;*

(ii) raises the marginal labour cost at H and lowers it at L: $d\omega_H/d\theta > 0 > d\omega_L/d\theta$;

(iii) shifts automation from L to H: $d\tilde{x}_H/d\theta > 0 > d\tilde{x}_L/d\theta$, *so the gap in automated shares widens;*

(iv) misallocates tasks in proportion to the displacement elasticity*: the gap in the shares of tasks that labour performs widens by*

$$\frac{d}{d\theta}\Big[\log(1-\tilde{x}_L)-\log(1-\tilde{x}_H)\Big] \;=\; h(\tilde{x}_H)\,\frac{d\log\omega_H}{d\theta} \;+\; h(\tilde{x}_L)\left|\frac{d\log\omega_L}{d\theta}\right| \;>\; 0. \quad (26)$$

The proof is in Online Appendix A.5.

Proposition 3 shows that introducing wage-setting power moves workers away from the more productive firm and widens the automation gap between firms. Starting from the equilibrium without labour markdowns, the productive firm's lower markdown reduces its relative posted wage, shifting workers towards L. With fewer workers, labour's marginal product rises at H and falls at L, and so does the marginal labour cost, which equals it at the optimum. Through the automation rule (24), H therefore automates more tasks and L fewer.

The displacement elasticity governs the size of these task distortions. Equation (26) weights each firm's change in marginal labour cost by its displacement elasticity. A flatter task schedule implies a higher elasticity, so the same cost change produces a larger distortion. Because the firms automate different shares of tasks, they face different elasticities. For similar cost changes, an increasing elasticity places more of the distortion at H, which automates too much; a decreasing elasticity places more at L, which automates too little.

3.3 From automation to labour market power

Markdowns depend on within-market employment shares, so cheaper automation changes wage-setting power if it reallocates workers across firms. Which firm gains workers depends on whose labour demand shifts out by more.

Write $z \equiv -d \log c_k > 0$ for the size of a marginal fall in the price of automation capital. Holding marginal labour cost and aggregate demand fixed, firm i's labour-demand response is

$$g(\tilde{x}_i) \equiv -\frac{\partial \log l_i^d}{\partial \log c_k}\bigg|_{\omega_i, Y} = \underbrace{\frac{\kappa_f}{\mu - \kappa_f}\tilde{x}_i}_{\text{scale effect}} - \underbrace{h(\tilde{x}_i)}_{\text{displacement effect}}. \tag{27}$$

Cheaper capital expands production while replacing workers in additional tasks. The difference between the firms' responses is

$$\Delta_\Upsilon \equiv g(\tilde{x}_H) - g(\tilde{x}_L). \tag{28}$$

I call the incidence of cheaper automation *concentrating* when $\Delta_\Upsilon > 0$ and *equalising* when $\Delta_\Upsilon < 0$.

Proposition 4. *After a marginal fall in the price of automation capital, with* Δ_Υ *from* (28)*:*

(i) both firms automate more*:* $d\tilde{x}_H > 0$ *and* $d\tilde{x}_L > 0$*;*

(ii) workers move towards the firm whose labour demand shifts out by more*:* $\text{sign}(ds_H) = \text{sign}(\Delta_\Upsilon)$*, so firm H gains employment share under concentrating incidence and loses it under equalising incidence, and shares do not change to first order if* $\Delta_\Upsilon = 0$*;*

(iii) wage-setting power follows the workers*: under concentrating incidence* δ_H *falls,* δ_L *rises, and the markdown gap* $\delta_L - \delta_H$ *widens; under equalising*

incidence these movements reverse.

The proof is in Online Appendix A.6.

Proposition 4 shows that a fall in the price of automation capital leads both firms to automate more tasks, because it raises each firm's marginal labour cost relative to that price, but can strengthen or weaken the more productive firm's wage-setting power. The outcome depends on the balance between scale and displacement.

Because H already automates more tasks, it benefits from a larger scale effect. With total employment fixed, it gains workers if this advantage exceeds its additional displacement, $\Delta_\Upsilon > 0$, where

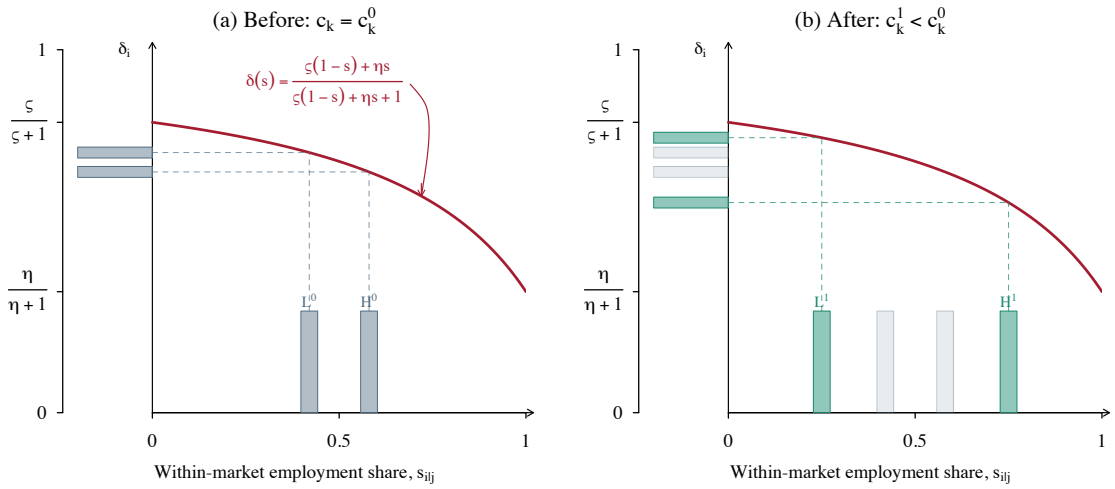
$$\Delta_\Upsilon = \underbrace{\frac{\kappa_f}{\mu - \kappa_f}\left(\tilde{x}_H - \tilde{x}_L\right)}_{\text{scale advantage of } H} - \underbrace{\left[h(\tilde{x}_H) - h(\tilde{x}_L)\right]}_{\text{displacement disadvantage of } H}. \tag{29}$$

What matters is the relative response: firm H can gain workers even if labour demand contracts at both firms, provided it contracts less at H.

The task schedule determines this balance. A constant or decreasing displacement elasticity makes incidence concentrating, since displacement cannot overturn H's scale advantage. An increasing elasticity makes incidence equalising only when H's additional displacement outweighs that advantage.

Wage-setting power follows the employment reallocation. Under concentrating incidence, H gains employment share and sets a lower markdown, while L's markdown rises. Under equalising incidence, these movements reverse. Cheaper automation therefore widens or narrows the markdown gap according to how displacement varies across the task schedule. Figure 1 illustrates the concentrating case.

Figure 1: EMPLOYMENT REALLOCATION AND MARKDOWN INCIDENCE



Notes: Superscripts 0 and 1 denote allocations before and after a fall in the automation price, and L and H denote the less and more productive firms. The red curve is the markdown schedule $\delta(s)$. Vertical bars show the firms' within-market employment shares; horizontal bars to the left of the vertical axis show the implied markdown distribution. Panel (a) shows the initial allocation. Panel (b) shows concentrating incidence: firm H gains employment share, firm L loses it, and the markdown gap widens. Pale bars reproduce the initial allocation. The markdown schedule is drawn with $\varsigma = 4$ and $\eta = 0.5$. Under equalising incidence, the employment-share movements reverse and the markdown gap narrows.

4 Extensions for the quantitative model

The quantitative model adds three elements to the baseline: heterogeneous installation costs, an extensive margin of automation, and physical capital. These let firms of the same size automate different shares of tasks, let some firms not automate at all, and align revenue in the model with value added in the data.

4.1 Heterogeneous installation costs

Installing automation equipment requires adaptation to firms' production processes, often through specialised integrators (Békés et al., 2025). I capture these costs with a firm-specific installation requirement $b_i > 0$, drawn from G_b. Installing one unit of automation capital requires b_i units of the automation-capital sector's

output, giving firm i the unit cost

$$c_{k,i} \;=\; c_k\, b_i. \tag{30}$$

The baseline first-order conditions therefore apply with c_k replaced by $c_{k,i}$. Automation depends on marginal labour cost relative to installation cost, $\omega_i/(c_k b_i)$. Firms with identical installation requirements retain the ranking of Proposition 2. A more productive firm can nevertheless automate less than a less productive firm if its installation requirement is sufficiently higher.

4.2 The extensive margin

The baseline task schedule implies that every firm automates, whereas more than half of French manufacturing firms have no recorded imports of automation capital. To allow non-adoption, I give the schedule a positive intercept, $R(0) > 0$. A firm adopts only when automation is cheaper than labour at the task best suited to capital:

$$\tilde{x}_i \;=\; \begin{cases} \dfrac{1}{M}\, R^{-1}\!\left(\dfrac{\omega_i}{c_k b_i}\right), & \omega_i > c_k\, b_i\, R(0), \\[2ex] 0, & \omega_i \le c_k\, b_i\, R(0). \end{cases} \tag{31}$$

The adoption indicator is $d_i = 1\{\tilde{x}_i > 0\}$. Among adopters, automation intensity increases with $\omega_i/(c_k b_i)$.

Proposition 5. *Labour market power can induce adoption that would not occur in the equilibrium without labour markdowns. Consider the analytical economy with $R(0) > 0$ and a common installation requirement b. If the productive firm is at the adoption threshold in the equilibrium without labour markdowns, $\omega_H = c_k b R(0)$, neither firm adopts. A sufficiently small introduction of wage-setting power induces the productive firm to adopt.*

The proof is in Online Appendix A.7.

Proposition 5 extends the task misallocation of Proposition 3 to the extensive margin. Wage-setting power raises the productive firm's marginal labour cost, and when that firm sits at the adoption threshold, the increase makes automation cheaper than labour at the first tasks, so it adopts where it would not without markdowns.

4.3 Physical capital

Firms combine the task composite F_i with physical capital k_i^p:

$$y_i \;=\; \phi_{ih}\underbrace{\left[\exp\!\left(\frac{1}{M}\int_0^M \log\!\big(M\, y_i(\tau)\big)\, d\tau\right)\right]}_{\equiv F_i}^{\kappa_f}\big(k_i^p\big)^{\kappa_p}, \qquad \kappa \;=\; \kappa_f+\kappa_p \;\in\; (0,1). \tag{32}$$

Physical capital enters outside the task composite, so it does not substitute for labour task by task, as opposed to automation capital.

A competitive producer supplies physical capital from the final good, with resource cost and supply price

$$G_p\big(K^p\big) \;=\; \frac{e}{1+e}\big(K^p\big)^{1+1/e}, \qquad p_p \;=\; \big(K^p\big)^{1/e}, \tag{33}$$

where $e>0$ is the supply elasticity.

Each firm spends the share κ_p/μ of revenue on physical capital. Summing over firms gives $p_pK^p=(\kappa_p/\mu)Y$, so the supply curve sets the price of physical capital as a function of aggregate output,

$$p_p \;=\; \left(\frac{\kappa_p}{\mu}\,Y\right)^{1/(1+e)}. \tag{34}$$

The capital producer's profit, $\Pi_p^U \equiv p_pK^p - G_p(K^p)$, accrues to firm owners in proportion to their profit shares at the welfare reference allocation and enters

their welfare. Aggregate consumption is

$$C = Y - \sum_i c_{k,i} k_i - G_p(K^p).$$

Supplemental Material S1.2 and Online Appendix B provide the extended model's equilibrium conditions and solution algorithm.

5 Quantification

This section takes the model to French manufacturing data. Section 5.1 describes the data and the measurement of automation. Section 5.2 documents that, within local labour markets, firms with more labour market power automate more, at both the extensive and the intensive margin. Section 5.3 sets out the parametric assumptions and normalisations, Section 5.4 the externally calibrated parameters, and Section 5.5 the simulated method of moments. Section 5.6 reports the estimates under three task schedules and the fit of the model.

5.1 Data

Sources. The data are a firm-year panel of French manufacturing firms from 2009 to 2019, built from four administrative sources that share the firm identifier. Balance sheets (FARE) give sales, value added, physical capital, intermediate inputs, employment, labour costs, location, and industry. The production survey (EAP) reports revenue and quantities by product for every firm with at least 20 employees or turnover above €5 million and for a representative sample of smaller firms; because it reports prices and quantities separately, it yields firm-level output prices. Workforce records (DADS Postes) report, for every job, the worker's wage, hours, age, occupation, and commuting zone, which I aggregate to the firm and year. Customs records report each firm's imports by product and year from 1996 to 2019, so every firm-year in the panel has a complete history of automation

imports.

Sample construction. I merge the balance sheets with the production survey and keep firms present in both, then add the workforce records and the Customs history. I drop five two-digit industries with very few observations or whose firms may be resellers of automation equipment, observations with missing prices, quantities, inputs, employment, or location, and firms with fewer than five employees on average, and I keep each firm's longest run of consecutive years, provided it lasts at least three years. Monetary variables are in 2015 euros. Revenue, prices, quantities, inputs, hours, employment and wages are winsorised at the first and ninety-ninth percentiles. The result is a panel of firms in eighteen manufacturing industries. I estimate production functions on the twelve of these industries in which automation capital is pervasive, which I call the production-function sample. A local labour market is a five-digit industry by commuting zone, employment is the firm's number of workers, and the within-market share $s_{i|j}$ is its share of the market's employment. Supplemental Material S4.1 provides additional details.

Automation. Automation is measured from imports. An import is automation capital when its product code is on the list of industrial robots, numerically controlled machines, automatic machine tools, and related equipment of Acemoglu and Restrepo (2022a). Let e_{it} denote firm i's imports of automation capital in year t, in 2015 euros. The automation stock accumulates these imports, depreciating each purchase linearly to zero over eight years,

$$K_{it}^{A} = \sum_{s=0}^{7} \left(1 - \frac{s}{8}\right) e_{i,t-s}, \tag{35}$$

so each purchase enters the stock at its full value, loses one eighth of that value each year, and leaves after eight years. A firm is an adopter in year t when $K_{it}^{A} > 0$, and its automation intensity is $K_{it}^{A}/\mathrm{VA}_{i,t-1}$, the stock per euro of lagged value

added. Supplemental Material S4.1.2 gives the product list.

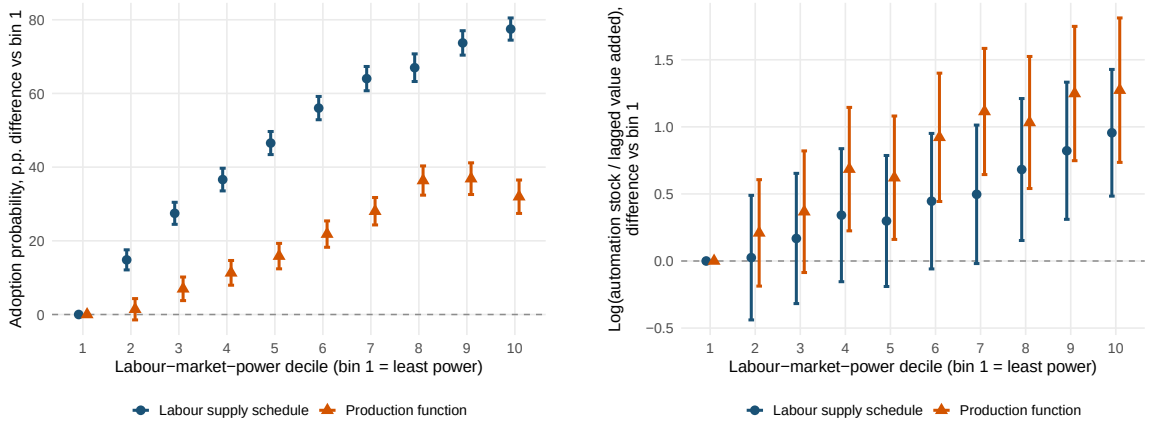
Sample statistics. The panel has 21,918 firms, 15,189 of them in 2019. The production-function sample has 16,525 firms and 134,762 firm-years and accounts for 86 per cent of the panel's sales. The production-function markdowns are estimated on the production-function sample; the model-implied markdowns, the market structure and the estimation moments use the whole panel. Automation is common but lumpy: in 2019, 44 per cent of the panel's firms hold a positive stock; in a typical year, about 30 per cent of firms in the production-function sample import automation capital; and the median firm, with 25 workers, imports none. Supplemental Material S4.1 reports the sample tables.

5.2 Labour market power and automation in the data

Labour market power. I measure wage markdowns in two ways. First, following Yeh et al. (2022), the production-function approach expresses the markdown as the labour share of revenue multiplied by the markup and divided by labour's output elasticity. I estimate the required output elasticities using translog production functions by two-digit industry.

Following Wong (2026), De Ridder et al. (2026) and De Loecker et al. (2016), I address three sources of bias in these estimates. A control function based on intermediate-input demand accounts for unobserved productivity. Firm-level output prices from the production survey allow me to estimate production functions in quantities, separating output-price variation from physical productivity. To account for unobserved input prices, I include sector-by-location fixed effects and a second control function that uses the firm's output price to proxy for input quality, on the premise that firms using higher-quality inputs charge higher prices. The estimated markdown has a median of 0.67 and a mean of 0.77, and declines with firm size. Supplemental Material S4.3 provides further details.

Figure 2: AUTOMATION AND LABOUR MARKET POWER



(a) Extensive margin

(b) Intensive margin

Notes: Estimates of β_b from regression (36), relative to bin 1, the highest markdowns; bin 10 has the most labour market power. Navy circles use the model-implied markdown $\delta(s_{i|j})$ on the whole panel; orange triangles use the production-function markdown of Supplemental Material S4 on the production-function sample. Panel (a): probability of holding a positive automation stock, in percentage points. Panel (b): log automation stock over lagged value added, in log points, adopters only. Standard errors are clustered by firm; the bands are 95 per cent confidence intervals. Both series cover 2010–2019 and keep market-year cells with at least two firms whose markdowns differ. Panel (a) uses 92,709 firm-years (14,266 firms) for the model-implied series and 70,777 (10,432 firms) for the production-function series; panel (b) uses 34,767 and 29,349 adopter firm-years.

Second, I compute the model-implied markdown, $\delta(s_{i|j})$, from each firm's employment share within its local labour market, using (13) and the labour-supply elasticities (ς, η) in Table 1.

Automation adoption and intensity. I sort firm-year observations into deciles of the markdown, from the least labour market power in bin 1, the highest markdowns, to the most in bin 10, and estimate, for the extensive and the intensive margin,

$$y_{it} \;=\; \sum_{b=2}^{10} \beta_b \, 1\{b_{it} = b\} \;+\; \gamma_{j(i)t} \;+\; \varepsilon_{it}, \tag{36}$$

where $\gamma_{j(i)t}$ is a local-labour-market by year fixed effect, so each β_b compares firms with competitors in the same market and year. For the extensive margin

the outcome is the adoption indicator. For the intensive margin the sample is restricted to adopters and the outcome is the log of the automation stock divided by lagged value added.[8]

Figure 2 reports the estimates. Both measures of the markdown give the same pattern: within local labour markets, the probability of holding automation capital and the automation intensity of adopters rise with labour market power. Using the model-implied (production-function) markdown to rank firms within each market and year, firms in the top decile are 77 (32) percentage points more likely to hold automation capital than those in the bottom decile. Among adopters, automation capital per euro of lagged value added is higher in the top decile than in the bottom decile by 0.96 (1.27) log points, about 2.6 (3.6) times as much.[9]

5.3 Parametric assumptions and normalisations

Task schedule. The estimation adopts the power–exponential family

$$\phi_l(\tau) = \zeta\Big(\frac{\tau}{M}\Big)^a \exp\Big(\beta_R \frac{\tau}{M}\Big), \qquad \phi_k(\tau) = \zeta\Big(\frac{\tau}{M}\Big)^a \Big(1 - \frac{\tau}{M}\Big)^a. \tag{37}$$

Here $a \in (0,1)$, $\beta_R > -a$, and $\zeta > 0$. Labour's comparative advantage is therefore $R(\tau) = \exp(\beta_R \tau/M)\,(1-\tau/M)^{-a}$. The restriction $\beta_R > -a$ makes R strictly increasing, with $R(M) = \infty$. The factor $(\tau/M)^a$ enters both productivities and cancels from R, so $R(0) = 1$, and a firm automates nothing when $\omega_i \le c_k b_i$.

In this family the displacement elasticity is $h(\tilde{x}) = [a + \beta_R(1-\tilde{x})]^{-1}$. A higher β_R makes labour's comparative advantage rise more steeply across tasks while leaving $R(0)$, and hence the adoption cutoff, unchanged. When $\beta_R = 0$, the specification

[8] Value added enters with a lag because contemporaneous value added is itself an outcome that automation affects. The bin boundaries are constructed once on the pooled sample and held fixed across the two margins.

[9] Omitting the automation stock from the production function may understate automation's effect on labour's output elasticity. Part of the lower labour share at automating firms is then attributed to a lower markdown, overstating the relationship between automation and wage-setting power. Model-implied markdowns do not use the production function and are unaffected.

reduces to the power specification of Guerreiro et al. (2022), with a constant displacement elasticity $1/a$. A given proportional fall in the automation price then displaces the same share of every adopter's remaining labour tasks, so the task schedule cannot generate equalising incidence between two adopters. A negative β_R makes the displacement elasticity decrease with automation intensity; a positive value makes it increase.

Distributional assumptions. Three firm-level primitives are drawn i.i.d. across firms. I assume that Hicks-neutral productivity is lognormal, $\log \phi_{ih} \sim N(\mu_\phi, \sigma_\phi^2)$, with dispersion σ_ϕ, and that amenities are Pareto, $T_i \sim \text{Pareto}(x_{\min}, \iota)$, with scale $x_{\min}$ and shape ι. I model the dependence between productivity and amenities with a Gaussian copula with correlation parameter $\mathcal{R}_{\phi T}$,

$$\begin{pmatrix} z_{\phi,i} \\ z_{T,i} \end{pmatrix} \sim N\left(0, \begin{pmatrix} 1 & \mathcal{R}_{\phi T} \\ \mathcal{R}_{\phi T} & 1 \end{pmatrix}\right), \quad \phi_{ih} = \exp(\mu_\phi + \sigma_\phi\, z_{\phi,i}), \quad T_i = x_{\min}\left[1 - F_N(z_{T,i})\right]^{-1/\iota}. \tag{38}$$

where F_N is the standard normal distribution function. The installation requirement has distribution G_b, with $\log b_i \sim N(\mu_b, \upsilon^2)$ and dispersion υ, independent of (ϕ_{ih}, T_i).

Normalisations. Every targeted moment is dimensionless, so the levels and units of several objects are not identified. Table 1 lists the parameters set by normalisation. Supplemental Material S3 provides further details.

5.4 Externally calibrated parameters

The labour-supply, product-demand, and technology parameters are calibrated externally. Following Azkarate-Askasua and Zerecero (2025), I set the between-market Fréchet shape to $\eta = 0.33$ and the within-market shape to $\varsigma = 3.455$, the upper end of their sector-specific estimates for French manufacturing. The demand

elasticity is $\rho = 4.333$, so that the CES markup is $\mu = \rho/(\rho - 1) = 1.3$, the lower end of the 1.3–1.6 range documented across OECD manufacturing by Calligaris et al. (2018). Total returns to scale, $\kappa = 0.972$, and the physical-capital elasticity, $\kappa_p = 0.338$, are also from Azkarate-Askasua and Zerecero's (2025) estimates for French manufacturing. The task-composite elasticity is the residual $\kappa_f = \kappa - \kappa_p = 0.634$, the net-revenue curvature is $\tilde{\mu} = \mu - \kappa_p = 0.962$, and the supply elasticity of physical capital is $e = 1.5$. The number of markets J, their firm counts N_j and the worker mass $\mathcal{L}$ are taken from the 2019 cross-section. Supplemental Material S4.1.4 provides further details.

Proposition 6. *Under the maintained identification assumptions, the parameters*

$$(\sigma_\phi, \iota, \mathcal{R}_{\phi T}, \upsilon, c_k, a, \beta_R)$$

are globally identified from the joint population distribution of wages, employment and revenue, together with the observed market structure.

The proof is in Online Appendix A.8.[10]

5.5 Simulated method of moments

Proposition 6 establishes identification from the full population distribution of observables. To assess how the quantitative implications depend on the shape of the displacement elasticity, I estimate the model separately under decreasing, constant and increasing elasticities, re-estimating firm heterogeneity and installation costs against the same empirical targets.

The baseline sets $a = 0.5$; Section 6.3 examines sensitivity to $a = 0.1$ and $a = 0.9$. The three baseline schedules set $\beta_R = -0.25$, 0 and 1, respectively. Conditional on

[10] For the identification of the within- and across-market Fréchet shape parameters, see Berger et al. (2022), Deb et al. (2024), Azkarate-Askasua and Zerecero (2025) and Felix (2026).

each schedule, I estimate $\boldsymbol{\psi} = (\sigma_\phi, \iota, \mathcal{R}_{\phi T}, \upsilon, c_k)$ by simulated method of moments:

$$\hat{\boldsymbol{\psi}}(a, \beta_R) = \arg\min_{\boldsymbol{\psi}} \big[\mathbf{m}^{\text{data}} - \mathbf{m}(\boldsymbol{\psi}; a, \beta_R)\big]' \mathbf{W} \big[\mathbf{m}^{\text{data}} - \mathbf{m}(\boldsymbol{\psi}; a, \beta_R)\big], \tag{39}$$

where $\mathbf{m}^{\text{data}}$ contains the five empirical moments, $\mathbf{m}(\boldsymbol{\psi}; a, \beta_R)$ their simulated counterparts, and $\mathbf{W}$ is the weighting matrix. The same moments and weighting matrix are used for every task schedule, including those of Section 6.3. Table 2 reports the five targeted moments and their simulated counterparts.

Measurement. The empirical moments use the 2019 cross-section. Wages are measured as each firm's deflated wage bill divided by its number of workers, and marginal labour costs are $\omega_i = w_i/\delta_i$, computed using the model-implied markdowns of Section 5.2. Supplemental Material S5.2 gives sample restrictions and moment formulas.

Firm heterogeneity. The first block disciplines productivity dispersion, σ_ϕ, amenity dispersion, ι, and the correlation between the two, $\mathcal{R}_{\phi T}$. More productive firms are larger and pay higher wages, whereas firms with better amenities attract more workers at lower wages. The within-market variances of log wages (M1) and log employment (M2), together with their covariance (M3), therefore distinguish productivity dispersion from amenity dispersion. Productivity moves wages and employment in the same direction, so productivity dispersion raises both variances and the covariance. Amenities move them in opposite directions, so amenity dispersion raises both variances but *lowers* the covariance. The correlation is identified because a productive firm that also offers good amenities reaches a large size with a smaller wage premium. A higher correlation therefore reduces wage dispersion relative to employment dispersion. Supplemental Material S5.1 provides further details.

Adoption selection. The second block disciplines the dispersion of installation requirements, υ. A firm adopts when $\omega_i > c_k b_i$. With a common installation requirement, adoption would follow a single cutoff in the marginal labour cost. With dispersed requirements, each firm has its own cutoff, $c_k b_i$: a firm with a low marginal labour cost adopts if its installation requirement is low enough, and a firm with a high marginal labour cost stays out if its requirement is high enough. Using the 2019 cross-section, I estimate the linear probability model for firms in markets with at least two firms:

$$d_i^{\text{data}} = \gamma_j + \beta_4 \log \omega_i^{\text{data}} + v_i, \tag{40}$$

where γ_j is a local-labour-market fixed effect, and target the slope $\hat{\beta}_4$ (M4). The slope measures how adoption rises with the marginal labour cost within a market. The more dispersed the installation requirements, the flatter it is.

Adoption rate. The third block disciplines the common capital cost, c_k, given the installation dispersion υ. A lower capital cost moves more firms across the adoption threshold. I therefore target the aggregate adoption rate (M5), defined as the share of firms in the 2019 panel that hold automation capital.

Weighting. I estimate the covariance matrix of the empirical moments, $\hat{\mathbf{C}}$, using 1,000 nonparametric bootstrap replications clustered by local labour market. The weighting matrix $\mathbf{W}$ is diagonal, with inverse bootstrap variances as weights, so the objective is the sum of squared discrepancies standardised by each moment's standard error. The full covariance matrix is used for inference.

5.6 Estimation results and model fit

Table 1 collects the model's parameters in three panels: those calibrated externally (Section 5.4), those set by normalisation (Section 5.3), and those estimated by SMM

Table 1: MODEL PARAMETERS

Externally calibrated			*Set by normalisation*		
ρ	Demand elasticity	4.333	M	Task measure	1
κ	Total returns to scale	0.972	$x_{\min}$	Amenity scale	$e^{-1/\iota}$
κ_p	Physical-capital elasticity	0.338	ζ	Task-schedule level	1
κ_f	Task-composite elasticity	0.634	$R(0)$	Comparative-advantage level	1
e	Supply elasticity of K^p	1.5	μ_ϕ	Productivity location	$-\sigma_\phi^2/2$
ς	Within-market shape	3.455	μ_b	Installation location	$-\upsilon^2/2$
η	Between-market shape	0.330	$\mathcal{L}$	Worker mass	Sample

		Displacement elasticity		
		Decreasing	Constant	Increasing
Fixed schedule parameters				
a	Task-schedule curvature	0.5	0.5	0.5
β_R	Task-schedule shape	−0.25	0	1
Estimated				
σ_ϕ	Productivity dispersion	0.577 (0.010)	0.567 (0.009)	0.546 (0.009)
ι	Amenity dispersion	0.755 (0.020)	0.755 (0.020)	0.754 (0.020)
$\mathcal{R}_{\phi T}$	Productivity–amenity correlation	0.363 (0.029)	0.375 (0.028)	0.403 (0.027)
υ	Installation dispersion	0.513 (0.034)	0.563 (0.039)	0.706 (0.090)
c_k	Automation capital price	0.957 (0.030)	1.075 (0.041)	1.694 (0.169)

Notes: The demand elasticity implies a markup of $\mu = 1.3$; $\kappa_f = \kappa - \kappa_p$ and $\tilde{\mu} = \mu - \kappa_p = 0.962$. The normalisations set $\mathbb{E}[\log T_i] = 0$, $\mathbb{E}[\phi_{ih}] = 1$, and $\mathbb{E}[b_i] = 1$; the copula (38) leaves the marginals and the normalisations unchanged. With $R(0) = 1$ and $\mathbb{E}[b_i] = 1$, a firm adopts when $\omega_i > c_k b_i$, and c_k is the average unit cost of installed automation capital; $\mathcal{L}$ is set equal to total employment in the estimation sample in 2019. The three schedules share $a = 0.5$ and differ in β_R; standard errors from (41) in parentheses.

under each of the three task schedules. Under each schedule the Jacobian of the five moments with respect to the five parameters has full rank, so the estimates are locally identified (Supplemental Material S5.1).

Standard errors. The estimated covariance matrix of $\hat{\boldsymbol{\psi}}$ is

$$\hat{\mathbf{V}} = (\hat{\mathbf{D}}'\mathbf{W}\hat{\mathbf{D}})^{-1}\hat{\mathbf{D}}'\mathbf{W}(\hat{\mathbf{C}} + \hat{\mathbf{C}}_{\text{sim}})\mathbf{W}\hat{\mathbf{D}}(\hat{\mathbf{D}}'\mathbf{W}\hat{\mathbf{D}})^{-1}, \tag{41}$$

where $\hat{\mathbf{D}}$ is the Jacobian of the five model moments with respect to the five estimated parameters, evaluated at $\hat{\boldsymbol{\psi}}$, $\mathbf{W}$ is the weighting matrix, and $\hat{\mathbf{C}}$ is the bootstrap

Table 2: TARGETED MOMENTS

			Model		
	Empirical target	Data	Decreasing	Constant	Increasing
(M1)	Within-market Var$(\log w_i)$	0.048 (0.003)	0.048	0.048	0.048
(M2)	Within-market Var$(\log l_i)$	0.765 (0.020)	0.765	0.765	0.765
(M3)	Within-market Cov$(\log w_i, \log l_i)$	0.020 (0.003)	0.020	0.020	0.020
(M4)	Within-market adoption slope, $\hat{\beta}_4$	0.438 (0.021)	0.438	0.438	0.438
(M5)	Aggregate adoption rate	0.437 (0.006)	0.437	0.437	0.437

Notes: The Data column reports the empirical moments computed on the 2019 cross-section of the panel, with a local labour market defined as a five-digit industry $\times$ commuting zone and an adopter defined by a positive eight-year automation stock. Standard errors, in parentheses, come from a nonparametric cluster bootstrap that draws local labour markets with replacement over 1,000 replications. The Model columns report the simulated counterparts at the estimated parameters of Table 1 under each task schedule.

covariance matrix of the empirical moments. The simulation covariance $\hat{\mathbf{C}}_{\text{sim}}$, estimated from 100 independent sets of firm draws at $\hat{\boldsymbol{\psi}}$, accounts for computing the model moments from a single simulated economy. The standard errors in Table 1 are the square roots of the diagonal elements of $\hat{\mathbf{V}}$.

Model fit on targeted moments. The model matches every targeted moment under each of the three task schedules (Table 2).

6 Counterfactual Experiments

I conduct two experiments, each under the decreasing-, constant- and increasing-elasticity schedules. Section 6.1 removes labour markdowns and compares the welfare gain with and without automation. Section 6.2 lowers the price of automation capital and asks how the welfare cost of labour market power changes. Section 6.3 shows that both findings survive when the model is re-estimated with $a = 0.1$ and $a = 0.9$, and with local labour markets defined by three-digit rather than five-digit industries within commuting zones.

6.1 The welfare cost of labour market power

I remove labour markdowns in two economies, setting $\delta_i = 1$ at every firm so that wages equal marginal revenue products. In the first, firms choose their automated shares before and after markdowns are removed. In the second, labour performs every task. The resulting welfare gain, expressed as a percentage of 2019 consumption, is my measure of the welfare cost of labour market power. I denote it by G in the economy with endogenous automation and by G^0 in the economy without automation.

Table 3 shows that removing markdowns raises social welfare by 6.3–9.0 per cent of 2019 consumption with endogenous automation, compared with 4.8–8.1 per cent without automation. Excluding automation therefore understates the welfare gain by 0.9–1.5 per cent of 2019 consumption, or 10–24 per cent of the gain with automation.

In both economies, removing markdowns raises wages by more at firms with greater wage-setting power and reallocates workers towards them. The reallocation is similar in the two economies, and so are the fall in consumption and the income transferred from owners to workers.[11]

The gain is larger with automation because the income that removing markdowns transfers to workers is worth more to them when firms can automate. Workers' welfare is their wage income times the employment index. The index is highest when employment is spread across firms in proportion to amenities, which is the allocation workers choose when all wages are equal, and it falls as wage dispersion moves employment away from that allocation (Supplemental

[11] Consumption falls because removing markdowns raises relative wages most in single-firm markets, where firms have greater wage-setting power. At the five-digit-industry-by-commuting-zone level, these markets account for 71 per cent of markets and around 48 per cent of employment (Figure S4.1). Firms in these single-firm markets have lower marginal revenue products of labour in the estimated economies, so the output lost through the movement of workers across markets outweighs the output gained from reallocation within markets, where workers move towards more productive firms with higher marginal revenue products of labour.

Table 3: THE WELFARE COST OF LABOUR MARKET POWER

	Decreasing elasticity		Constant elasticity		Increasing elasticity	
Automation	With	Without	With	Without	With	Without
Panel A. Oligopsonistic economy, levels with the 2019 economy = 1						
Consumption C	1	0.513	1	0.563	1	0.719
Wage bill $\mathcal{B}$, share of 2019 C	0.130	0.124	0.144	0.136	0.186	0.174
Workers' welfare $\widetilde{\mathcal{W}}$ (2019 = 1)	1	0.902	1	0.905	1	0.926
Employment index $\mathcal{N}/\bar{\mathcal{N}}$	1	0.947	1	0.959	1	0.988
Owners' welfare $\mathcal{W}^f$ (2019 = 1)	1	0.447	1	0.499	1	0.669
Social welfare $\mathcal{S}$	1	0.506	1	0.558	1	0.717
Panel B. Equilibrium without labour markdowns, levels with the 2019 economy = 1						
Consumption C	0.994	0.507	0.993	0.556	0.990	0.710
Wage bill $\mathcal{B}$, share of 2019 C	0.309	0.293	0.343	0.321	0.439	0.410
Workers' welfare $\widetilde{\mathcal{W}}$ (2019 = 1)	2.904	2.615	2.903	2.623	2.899	2.682
Employment index $\mathcal{N}/\bar{\mathcal{N}}$	1.221	1.162	1.223	1.178	1.228	1.215
Owners' welfare $\mathcal{W}^f$ (2019 = 1)	0.787	0.246	0.760	0.274	0.677	0.368
Social welfare $\mathcal{S}$	1.062	0.554	1.070	0.614	1.090	0.798
Welfare cost G, G^0 (% of 2019 C)	6.25	4.77	6.95	5.58	9.00	8.11
Difference $G - G^0$ (% of 2019 C)		1.47		1.37		0.89
Difference (% of G)		23.6		19.7		9.9

Notes: Each pair of columns is the economy estimated under the stated displacement elasticity. In the *With* column, firms choose their automated shares; in the *Without* column, every automated share is held at zero. G is the welfare cost in the *With* economy and G^0 in the *Without* economy. Levels are relative to the estimated 2019 economy, with the welfare reference fixed at that economy. The wage bill is expressed as a share of 2019 consumption, so that the two economies are compared in the same unit. Workers' and owners' welfare are each normalised by their respective 2019 levels. Workers' welfare is their consumption equivalent, $\widetilde{\mathcal{W}} \equiv \sigma C \mathcal{N}/\bar{\mathcal{N}}$, the first term of (22). The employment index $\mathcal{N}/\bar{\mathcal{N}}$ is workers' welfare divided by the wage bill, both relative to the 2019 economy. Social welfare is measured by (22). The welfare cost is the change in social welfare from Panel A to Panel B, and the difference is $G - G^0$. Differences and percentage changes are computed from unrounded values.

Material S1.1). Without automation, producing at a more productive firm requires workers there, so concentrating production in those firms also concentrates employment and pulls it further from the allocation proportional to amenities. With automation, more productive firms perform part of their tasks with automation capital, and the more productive the firm the larger that part (Proposition 2), so production can concentrate in those firms without concentrating employment.

The employment index is therefore higher with automation, before and after markdowns are removed, and each unit of income that removing markdowns transfers to workers adds more to welfare.[12]

6.2 Welfare effects of cheaper automation

I next ask whether a fall in the price of automation capital raises the welfare cost of labour market power.[13] The welfare cost at automation price c_k is the gap between social welfare without and with labour markdowns, $G(c_k) \equiv 100\,[\mathcal{S}^{\mathrm{B}}(c_k) - \mathcal{S}(c_k)]/\bar{C}$, where $\mathcal{S}(c_k)$ is social welfare (22) in the oligopsonistic economy at automation price c_k, the superscript B marks the equilibrium without labour markdowns, and $\bar{C}$ is 2019 consumption. At the 2019 price, $G(c_k)$ is the gain G of Table 3. I lower the price in both economies and compare $G(c_k)$ before and after. The welfare cost of labour market power rises when cheaper automation raises welfare by more without labour markdowns than with them.

Table 4 reports the results.[14] Cheaper automation raises social welfare by 15 to 29 per cent of 2019 consumption in the oligopsonistic economy. The price reduction lowers the cost of performing tasks already automated and makes automation profitable for additional tasks. Firms expand production, and consumption rises by 15 to 29 per cent: the economy produces more of the final good with the same workforce.

[12] Table 3 reports an employment index of 1 with automation and 0.95–0.99 without automation in the estimated 2019 economies. Removing markdowns raises these values to 1.22–1.23 and 1.16–1.21, respectively. The increase is approximately 0.22 in both cases. The larger welfare gain with automation, $G > G^0$, therefore reflects the higher value to workers of each unit of income transferred to them, rather than a larger improvement in the employment index.

[13] The size of the fall is set so that aggregate automation capital in the oligopsonistic economy grows by 80 per cent, the projected growth in the number of industrial robots in France from 2019 to 2029 (Supplemental Material S5.3). Each estimated economy therefore has its own cut, between 12.5 and 14.8 per cent. The target is growth in industrial robots, whereas automation capital is broader, so the size of the cut is conditional on this proxy.

[14] Because the cut is set separately for each schedule to deliver the same growth in automation capital, differences across columns reflect both the shape of the task schedule and the size of the cut.

Table 4: CHEAPER AUTOMATION AND THE WELFARE COST OF LABOUR MARKET POWER

	Displacement elasticity		
	Decreasing	Constant	Increasing
Automation-price reduction (%)	12.5	12.9	14.8
Increase in consumption, oligopsonistic economy (%)	28.95	25.61	15.49
Welfare cost of labour market power (% of 2019 C)			
At the 2019 price	6.25	6.95	9.00
At the lower price	6.52	7.25	9.28
Increase (% of 2019 C)	0.27	0.30	0.28
Increase (% of the cost at the 2019 price)	4.4	4.3	3.1
Gain from cheaper automation (% of 2019 C)			
Oligopsonistic economy: social welfare $\mathcal{S}$	29.03	25.68	15.49
Workers' welfare $\widetilde{\mathcal{W}}$	0.37	0.46	0.60
Owners' welfare $\mathcal{W}^f$	28.66	25.22	14.89
No labour markdowns: social welfare $\mathcal{S}$	29.30	25.98	15.77
Workers' welfare $\widetilde{\mathcal{W}}$	1.09	1.34	1.76
Owners' welfare $\mathcal{W}^f$	28.21	24.64	14.01

Notes: Each gain from cheaper automation compares an economy at the lower price with the same economy at the 2019 price. Workers' and owners' gains are changes in $\widetilde{\mathcal{W}}$ and $\mathcal{W}^f$, in per cent of 2019 consumption, and sum to the social welfare gain. Welfare uses the 2019 economy as reference. Differences and percentage changes are computed from unrounded values.

Owners receive almost all of the welfare gain. Workers gain 0.4 to 0.6 per cent of 2019 consumption, while owners gain 15 to 29 per cent. Cheaper automation expands production, but it also replaces workers in additional tasks, limiting the increase in labour demand and wages. Firms can therefore produce substantially more without a comparable increase in the aggregate wage bill, and much of the benefit accrues as profit. Labour market power further limits workers' gains: without markdowns, their welfare gain is about three times as large.

Cheaper automation also raises the welfare cost of labour market power by 0.27 to 0.30 per cent of 2019 consumption, or 3 to 4 per cent of its initial level. Proposition 4 identifies how cheaper automation can change the underlying distortion. Firms that already automate more benefit from a larger reduction in unit costs, giving them a stronger incentive to expand. If this scale advantage outweighs

their additional displacement of labour, they attract workers and acquire greater wage-setting power, widening the welfare gap between the economies with and without labour markdowns.

6.3 Sensitivity analysis

I re-estimate the model with the curvature of the task schedule set to $a = 0.1$ and $a = 0.9$ instead of 0.5, under each shape of the displacement elasticity. Across these and the baseline estimates, ignoring automation understates the gain from removing markdowns by 4–48 per cent, and cheaper automation raises the welfare cost by 1.0–5.2 per cent. With local labour markets defined as three-digit rather than five-digit industries within commuting zones, the two ranges are 8–20 and 2.8–4.0 per cent. Supplemental Material S6 provides additional details.

7 Conclusion

I develop and estimate a quantitative general equilibrium model of oligopsonistic labour markets with endogenous task-based automation, and provide conditions under which its equilibrium is unique and its parameters are globally identified. The model nests the competitive task-based model and the oligopsony model without automation as special cases.

Two mechanisms make automation and labour market power interact. First, wage-setting power puts a wedge between a firm's marginal labour cost and its wage. The wedge raises the cost of labour relative to automation capital and so shapes which firms automate and how much. Second, automation changes a firm's size relative to its local labour market and hence its wage-setting power. Labour's comparative advantage across tasks governs both links.

The quantitative analysis shows that the interaction between automation and labour market power matters for welfare. Across the three specifications

of labour's comparative advantage, excluding automation understates the gains from removing labour market power by 10–24 per cent. Cheaper automation raises social welfare, delivers most of its gains to firm owners and increases the welfare cost of labour market power by 3–4 per cent of its initial level.

Accounting for the interaction between automation and labour market power matters for evaluating both policies that address employer power and policies that redistribute income. Minimum wages (Berger et al., 2025) and collective bargaining (Azkarate-Askasua and Zerecero, 2025) can raise marginal labour costs and induce automation, potentially limiting gains to workers. Progressive taxation in an oligopsonistic economy can exacerbate misallocation by reallocating workers away from more productive firms (Berger et al., 2024), but automation may cushion the resulting output loss by allowing these firms to substitute automation capital for workers. The framework provides a basis for assessing these policy effects, as well as whether automation of new tasks through AI (Althoff and Reichardt, 2026) strengthens or weakens employer power.

References

[1] **Acemoglu, D., and D. Autor.** 2011. "Skills, Tasks and Technologies: Implications for Employment and Earnings." In *Handbook of Labor Economics*, Volume 4. 1043–1171, Elsevier.

[2] **Acemoglu, D., and P. Restrepo.** 2018. "The Race Between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment." *American Economic Review* 108 (6): 1488–1542.

[3] **Acemoglu, D., and P. Restrepo.** 2019. "Automation and New Tasks: How Technology Displaces and Reinstates Labor." *Journal of Economic Perspectives* 33 (2): 3–30. DOI.

[4] **Acemoglu, D., and P. Restrepo.** 2020. "Robots and Jobs: Evidence From US Labor Markets." *Journal of Political Economy* 128 (6): 2188–2244.

[5] **Acemoglu, D., and P. Restrepo.** 2022a. "Demographics and Automation." *The Review of Economic Studies* 89 (1): 1–44.

[6] **Acemoglu, D., and P. Restrepo.** 2022b. "Tasks, Automation, and the Rise in US Wage Inequality." *Econometrica* 90 (5): 1973–2016.

[7] **Acemoglu, D., and P. Restrepo.** 2026. "Automation and Rent Dissipation: Implications for Wages, Inequality, and Productivity." *Quarterly Journal of Economics* 141 (2): 1521–1579.

[8] **Aghion, P., C. Antonin, S. Bunel, and X. Jaravel.** 2023. "Modern Manufacturing Capital, Labor Demand and Product Market Dynamics: Evidence from France." CEP Discussion Paper 1910, Centre for Economic Performance, London School of Economics.

[9] **Althoff, L., and H. Reichardt.** 2026. "Task-Specific Technical Change and Comparative Advantage." Working Paper 35353, National Bureau of Economic Research. DOI.

[10] **Amodio, F., P. Medina, and M. Morlacco.** 2025. "Labor Market Power, Self-Employment, and Development." *American Economic Review* 115 (9): 3014–3057. DOI.

[11] **Armangué-Jubert, T., N. Guner, and A. Ruggieri.** 2025. "Labor Market Power and Development." *American Economic Review: Insights* 7 (2): 177–195. DOI.

[12] **Azar, J. A., S. T. Berry, and I. Marinescu.** 2022. "Estimating Labor Market Power." Working Paper 30365, National Bureau of Economic Research. DOI.

[13] **Azar, J., M. Chugunova, K. Keller, and S. Samila.** 2024. "Monopsony and Automation." Research Paper 23-21, Max Planck Institute for Innovation and Competition.

[14] **Azkarate-Askasua, M., and M. Zerecero.** 2025. "Union and Firm Labor Market Power." Unpublished manuscript, SSRN Working Paper 4323492, June 2025 version, Website.

[15] **Békés, G., A. Bonfiglioli, R. Crinò, and G. Gancia.** 2025. "Integrators and Robot

Adoption: Facts From Hungary." CEPR Discussion Paper 20026.

[16] **Berger, D., K. Herkenhoff, and S. Mongey.** 2022. "Labor Market Power." *American Economic Review* 112 (4): 1147–1193.

[17] **Berger, D., K. Herkenhoff, and S. Mongey.** 2025. "Minimum Wages, Efficiency, and Welfare." *Econometrica* 93 (1): 265–301.

[18] **Berger, D., K. Herkenhoff, S. Mongey, and N. Mousavi.** 2024. "Monopsony Amplifies Distortions from Progressive Taxes." *AEA Papers and Proceedings* 114: 555–560. DOI.

[19] **Bonfiglioli, A., R. Crinò, H. Fadinger, and G. Gancia.** 2024. "Robot Imports and Firm-Level Outcomes." *The Economic Journal* 134 (664): 3428–3444.

[20] **Brynjolfsson, E., J. F. Li, J. Miranda, R. Seamans, and A. Wang.** 2026. "Minimum Wages and the Rise of the Robots." CES Working Paper 26-42, U.S. Census Bureau, Center for Economic Studies.

[21] **Byambasuren, T.** 2026. "Automation Threat and Labor Market Power." Unpublished manuscript, SSRN Working Paper 7014598, June 2026 version.

[22] **Calligaris, S., C. Criscuolo, and L. Marcolin.** 2018. "Mark-Ups in the Digital Era." OECD Science, Technology and Industry Working Papers 2018/10, OECD Publishing.

[23] **Card, D., A. R. Cardoso, J. Heining, and P. Kline.** 2018. "Firms and Labor Market Inequality: Evidence and Some Theory." *Journal of Labor Economics* 36 (S1): S13–S70.

[24] **Dauth, W., S. Findeisen, J. Suedekum, and N. Woessner.** 2021. "The Adjustment of Labor Markets to Robots." *Journal of the European Economic Association* 19 (6): 3104–3153. DOI.

[25] **De Loecker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik.** 2016. "Prices, Markups, and Trade Reform." *Econometrica* 84 (2): 445–510.

[26] **De Ridder, M., B. Grassi, and G. Morzenti.** 2026. "The Hitchhiker's Guide to

Markup Estimation: Assessing Estimates From Financial Data." *Econometrica* 94 (1): 137–168.

[27] **Deb, S., J. Eeckhout, A. Patel, and L. Warren.** 2022. "What Drives Wage Stagnation: Monopsony or Monopoly?" *Journal of the European Economic Association* 20 (6): 2181–2225.

[28] **Deb, S., J. Eeckhout, A. Patel, and L. Warren.** 2024. "Walras–Bowley Lecture: Market Power and Wage Inequality." *Econometrica* 92 (3): 603–636.

[29] **Eden, M., and P. Gaggl.** 2018. "On the Welfare Implications of Automation." *Review of Economic Dynamics* 29: 15–43.

[30] **Felix, M.** 2026. "Trade, Labor Market Concentration, and Wages." Working Paper 35018, National Bureau of Economic Research.

[31] **Firooz, H., Z. Liu, and Y. Wang.** 2025. "Automation and the Rise of Superstar Firms." *Journal of Monetary Economics* 151: 103733.

[32] **Guerreiro, J., S. Rebelo, and P. Teles.** 2022. "Should Robots Be Taxed?" *The Review of Economic Studies* 89 (1): 279–311.

[33] **Gutiérrez, A.** 2023. "Labor Market Power and the Pro-Competitive Gains from Trade." Working Paper, University of Chicago.

[34] **Hubmer, J., and P. Restrepo.** 2026. "Not a Typical Firm: Capital–Labor Substitution and Firms' Labor Shares." *American Economic Journal: Macroeconomics* 18 (2): 34–71.

[35] **Humlum, A.** 2021. "Robot Adoption and Labor Market Dynamics." Unpublished manuscript, University of Chicago, November 2021 version.

[36] **Hurst, E., P. J. Kehoe, E. Pastorino, and T. Winberry.** 2025. "The Macroeconomic Dynamics of Labor Market Policies." *Journal of Political Economy*, Forthcoming.

[37] **Jaimovich, N., I. Saporta-Eksten, H. Siu, and Y. Yedid-Levi.** 2021. "The Macroeconomics of Automation: Data, Theory, and Policy Analysis." *Journal of Monetary Economics* 122: 1–16.

[38] **Jarosch, G., J. S. Nimczik, and I. Sorkin.** 2024. "Granular Search, Market Structure, and Wages." *Review of Economic Studies* 91 (6): 3569–3607. DOI.

[39] **Kikuchi, S.** 2023. "Labor Market Concentration, Automation, and Labor Share." Unpublished manuscript, MIT, July 2023 version, Website.

[40] **Koch, M., I. Manuylov, and M. Smolka.** 2021. "Robots and Firms." *The Economic Journal* 131 (638): 2553–2584.

[41] **Lamadon, T., M. Mogstad, and B. Setzler.** 2022. "Imperfect Competition, Compensating Differentials, and Rent Sharing in the US Labor Market." *American Economic Review* 112 (1): 169–212.

[42] **Leduc, S., and Z. Liu.** 2024. "Automation, Bargaining Power, and Labor Market Fluctuations." *American Economic Journal: Macroeconomics* 16 (4): 311–349.

[43] **Lehr, N. H.** 2026. "Does Monopsony Matter for Innovation?" Working paper, International Monetary Fund.

[44] **Moll, B., L. Rachel, and P. Restrepo.** 2022. "Uneven Growth: Automation's Impact on Income and Wealth Inequality." *Econometrica* 90 (6): 2645–2683.

[45] **Rubens, M.** 2025. "Labor-Market Power, Deadweight Loss, and Technology Adoption." NBER Working Paper 30586, National Bureau of Economic Research.

[46] **Stiebale, J., J. Suedekum, and N. Woessner.** 2024. "Robots and the Rise of European Superstar Firms." *International Journal of Industrial Organization* 97: 103085.

[47] **vom Lehn, C.** 2020. "Labor Market Polarization, the Decline of Routine Work, and Technological Change: A Quantitative Analysis." *Journal of Monetary Economics* 110: 62–80.

[48] **Wong, H. C.** 2026. "Understanding High-Wage Firms: Monopoly, Monopsony, and Bargaining Power." *American Economic Review* 116 (7): 2504–2541.

[49] **Yeh, C., C. Macaluso, and B. Hershbein.** 2022. "Monopsony in the US Labor Market." *American Economic Review* 112 (7): 2099–2138.

ONLINE APPENDIX

LABOUR MARKET POWER, AUTOMATION AND WELFARE

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5 November 2026

This Online Appendix is organised as follows. Appendix A proves the propositions of the main text. Appendix B gives the algorithm that solves the quantitative model. Equations, propositions, sections, tables and figures without a letter prefix refer to the main paper; those with an S prefix refer to the Supplemental Material.

A Proofs

A.1 Assumptions and lemmas

This section states a maintained assumption and two lemmas used throughout the proofs.

Maintained assumption.

(A1) Each firm takes Y and Φ as given, as in Definition 1(ii), including in the comparative statics, in which Y and Φ move but no firm internalises its own effect on them. In the analytical economy the single market employs every worker. A firm that internalised this would recognise that raising its wage cannot draw workers from other markets, and the across-market term in (6) would vanish. I assume instead that each firm takes Φ as given, as in the quantitative model, so the elasticity it perceives is (6) and its markdown is the same function of its share as there.

The lemmas are stated for the baseline model of Section 2, in which the comparative-advantage schedule starts at zero and every firm automates some, but not all, tasks. The final paragraph of this section extends them to a positive intercept and to the adoption margin of Section 4.2.

Notation. Write $\tilde{R}(\tilde{x}) \equiv R(M\tilde{x})$ for the schedule in normalised units. In terms of its slope, the displacement elasticity (16) and the direct labour-demand shift (27) are

$$\psi(\tilde{x}) \equiv \frac{d \log \tilde{R}(\tilde{x})}{d\tilde{x}} > 0, \qquad h(\tilde{x}) = \frac{1}{(1-\tilde{x})\,\psi(\tilde{x})}, \qquad g(\tilde{x}) = \frac{\kappa_f\,\tilde{x}}{\mu - \kappa_f} - h(\tilde{x}).$$

For firm i, $h_i \equiv h(\tilde{x}_i)$ and $g_i \equiv g(\tilde{x}_i)$. Since $\varepsilon'(s) = -(\varsigma - \eta) < 0$ because $\eta < \varsigma$, the markdown map (13) is strictly decreasing,

$$\delta'(s) = -\frac{\varsigma - \eta}{\big(1 + \varepsilon(s)\big)^2} < 0. \tag{A.1}$$

Tasks.

Lemma 1 (Task assignment and the automation threshold)**.** *Let Assumption 1 hold and fix factor prices $c > 0$ for automation capital and $\omega > 0$ for labour.*

(i) *Any cost-minimising assignment of tasks to factors uses a single factor at almost every task and takes a threshold form: there exists a unique $x \in (0, M)$ such that tasks $\tau < x$ use capital only and tasks $\tau > x$ use labour only, with $x = R^{-1}(\omega/c)$.*

(ii) *The automated share $\tilde{x} = x/M \in (0, 1)$ depends on prices only through ω/c, is invariant to Hicks-neutral productivity ϕ_h, and is continuously differentiable and strictly increasing in ω/c, with*

$$d\tilde{x} = (1-\tilde{x})\, h(\tilde{x}) \left(d\log\omega - d\log c\right), \qquad d\log(1-\tilde{x}) = -\,h(\tilde{x}) \left(d\log\omega - d\log c\right). \tag{A.2}$$

Proof. (i) Every task is essential under the logarithmic aggregate in (3), so cost minimisation requires $y(\tau) > 0$ at almost every task, and minimises cost task by task: producing one unit of task output costs $c/\phi_k(\tau)$ with capital and $\omega/\phi_l(\tau)$ with labour, and since task output is linear in the two inputs, the minimal cost is $\min\{c/\phi_k(\tau),\, \omega/\phi_l(\tau)\}$, attained by using only the cheaper factor. Capital is strictly cheaper at task τ if and only if $c/\phi_k(\tau) < \omega/\phi_l(\tau)$, that is, $R(\tau) < \omega/c$. Under Assumption 1, R is continuous and strictly increasing on $(0, M)$ with $R(\tau) \to 0$ as $\tau \to 0$ and $R(\tau) \to \infty$ as $\tau \to M$, so the set of tasks at which capital is strictly cheaper is the non-empty interval $[0, x)$ with $x = R^{-1}(\omega/c)$. Strict monotonicity of R makes the threshold unique, and indifference between the factors occurs at

most at the single task $\tau = x$, a set of measure zero. Because $R(\tau) \to \infty$ as $\tau \to M$, labour is strictly cheaper on a neighbourhood of M, so $x < M$.

(ii) Part (i) gives $\tilde{x} = M^{-1}R^{-1}(\omega/c)$, a function of ω/c alone, so ϕ_h does not enter, and $R \in C^1$ with $R' > 0$ gives continuous differentiability and strict monotonicity. Taking logs of $\tilde{R}(\tilde{x}) = \omega/c$ and differentiating, $\psi(\tilde{x})\, d\tilde{x} = d\log\omega - d\log c$, and $1/\psi(\tilde{x}) = (1-\tilde{x})\, h(\tilde{x})$ by the definition of the displacement elasticity. Dividing by $-(1-\tilde{x})$ gives the second expression in (A.2). □

The lemma is stated for arbitrary factor prices, so it applies at each firm's prices (c_k, ω_i), where the marginal labour cost $\omega_i = w_i + \xi_i$ includes the endogenous shadow value of the labour-supply constraint: whatever wage the firm posts, its cost-minimising task assignment has the threshold form (4) used in the main text.

Costs and demands at given factor prices. Throughout, the task productivities satisfy

$$\int_0^M \left(|\log\phi_k(\tau)| + |\log\phi_l(\tau)|\right) d\tau < \infty, \tag{A.3}$$

so the unit cost (A.5) below is finite.

Lemma 2 (Costs, output, revenue, and labour demand at given factor prices)**.** *Fix factor prices (c_k, ω) and productivity ϕ_h, and let $\tilde{x} = \tilde{x}(\omega/c_k)$ be the threshold of Lemma 1. Write $\varrho \equiv (\mu-1)/(\mu-\kappa_f) \in (0,1)$ and $\beta \equiv \kappa_f\mu/(\mu-\kappa_f) > 0$* (local to this appendix)*.*

(i) Costs. *The cost function, the minimum cost of producing output y when labour is priced at its marginal labour cost ω rather than at the posted wage, is*

$$TC(y;\, c_k, \omega, \phi_h) = \left(\frac{y}{\phi_h}\right)^{1/\kappa_f} q(c_k, \omega), \tag{A.4}$$

where the unit cost of the task bundle is the geometric mean across tasks of

the cheaper factor's cost per efficiency unit,

$$\log q(c_k, \omega) \equiv \int_0^{\tilde{x}} \log \frac{c_k}{\phi_k(Mu)}\, du + \int_{\tilde{x}}^1 \log \frac{\omega}{\phi_l(Mu)}\, du. \tag{A.5}$$

Shephard's lemma gives

$$\frac{\partial \log q}{\partial \log \omega} = 1 - \tilde{x} \in (0,1), \qquad \frac{\partial \log q}{\partial \log c_k} = \tilde{x} \in (0,1). \tag{A.6}$$

(ii) Output and revenue. *Given Y, the output that maximises $Y^{1/\rho} y^{1/\mu} - TC(y)$ is unique and log-linear in the primitives,*

$$\log y = \beta \left[\log \frac{\kappa_f}{\mu} + \frac{1}{\rho} \log Y + \frac{1}{\kappa_f} \log \phi_h - \log q(c_k, \omega) \right], \tag{A.7}$$

with revenue $\log r = \frac{1}{\rho} \log Y + \frac{1}{\mu} \log y$ *and elasticities*

$$\begin{gathered} \frac{\partial \log r}{\partial \log \phi_h} = \frac{1}{\mu - \kappa_f}, \quad \frac{\partial \log r}{\partial \log \omega} = -\frac{\kappa_f\,(1-\tilde{x})}{\mu - \kappa_f}, \\ \frac{\partial \log r}{\partial \log c_k} = -\frac{\kappa_f\,\tilde{x}}{\mu - \kappa_f}, \quad \frac{\partial \log r}{\partial \log Y} = \varrho. \end{gathered} \tag{A.8}$$

(iii) Labour demand. *The labour demand l^d, defined by $\omega\, l^d = (\kappa_f/\mu)(1-\tilde{x})\, r$ as in (10), satisfies*

$$\frac{\partial \log l^d}{\partial \log \phi_h} = \frac{1}{\mu - \kappa_f}, \quad \frac{\partial \log l^d}{\partial \log \omega} = -\mathcal{F}(\tilde{x}), \quad \frac{\partial \log l^d}{\partial \log c_k} = -g(\tilde{x}), \quad \frac{\partial \log l^d}{\partial \log Y} = \varrho, \tag{A.9}$$

where

$$\mathcal{F}(\tilde{x}) \equiv \underbrace{h(\tilde{x})}_{\text{task displacement}} + \underbrace{\frac{\kappa_f\,(1-\tilde{x})}{\mu - \kappa_f}}_{\text{scale}} + \underbrace{1}_{\text{factor intensity}} > 1, \tag{A.10}$$

$$\mathcal{F}(\tilde{x}) + g(\tilde{x}) = \frac{\mu}{\mu - \kappa_f} = \rho\,\varrho. \tag{A.11}$$

(iv) Equilibrium consistency. *Evaluated at the firm's marginal labour cost and productivity* (ω_i, ϕ_{ih})*, the solution of parts (i)–(iii) satisfies the threshold* (14)*, the factor demands* (10)*, and* $\text{MRPL}_i = \omega_i$*; hence in equilibrium* $l_i = l^d(\omega_i; \phi_{ih})$*,* $r_i = r(\omega_i; \phi_{ih})$*,* $y_i = y(\omega_i; \phi_{ih})$*.*

(v) Global optimality. *Along the labour supply curve the firm perceives,* (1) *with rivals' wages,* Φ*, and* Y *held fixed per (A1), profit is strictly quasi-concave in the posted wage, so the wage that solves the first-order condition is the unique global optimum.*

Proof. (i) Let Q denote the bracketed task aggregate in (3), so that $y = \phi_h Q^{\kappa_f}$ and $Q = (y/\phi_h)^{1/\kappa_f}$. By Lemma 1(i), one unit of task output at τ costs the cheaper factor's cost per efficiency unit, $m(\tau)$, so the firm chooses $\{y(\tau)\}$ to solve

$$\min_{\{y(\tau)\}} \int_0^M m(\tau)\, y(\tau)\, d\tau \qquad \text{s.t.} \qquad \frac{1}{M}\int_0^M \log\big(M y(\tau)\big)\, d\tau = \log Q,$$
$$m(\tau) \equiv \min\Big\{\frac{c_k}{\phi_k(\tau)}, \frac{\omega}{\phi_l(\tau)}\Big\}.$$

The first-order condition $m(\tau)\, y(\tau) = \lambda/M$ equalises expenditure across tasks, with the multiplier λ equal to total cost. Substituting $M y(\tau) = \lambda/m(\tau)$ into the constraint and solving for the multiplier,

$$\lambda = Q \exp\Big\{\int_0^1 \log m(Mu)\, du\Big\} = Q\, q(c_k, \omega),$$

with q as in (A.5) because the minimum inside m selects capital below the threshold and labour above it. This is (A.4). For (A.6), differentiate $\log q$: the direct price arguments contribute $(1-\tilde{x})\, d\log\omega + \tilde{x}\, d\log c_k$, and the movement of the limit $\tilde{x}$ contributes $[\log(c_k/\phi_k(M\tilde{x})) - \log(\omega/\phi_l(M\tilde{x}))]\, d\tilde{x} = 0$, because the two costs coincide at the threshold.

(ii) The objective $Y^{1/\rho} y^{1/\mu} - (y/\phi_h)^{1/\kappa_f} q(c_k, \omega)$ is strictly concave in y (revenue strictly concave since $1/\mu < 1$; cost strictly convex since $1/\kappa_f > 1$). The first-order

condition,

$$\frac{1}{\mu}\, Y^{1/\rho}\, y^{1/\mu-1} = \frac{1}{\kappa_f}\, \phi_h^{-1/\kappa_f}\, y^{1/\kappa_f-1}\, q(c_k,\omega),$$

collects to $y^{1/\kappa_f-1/\mu} = (\kappa_f/\mu)\, Y^{1/\rho}\phi_h^{1/\kappa_f} q(c_k,\omega)^{-1}$ with $1/\kappa_f - 1/\mu = 1/\beta$, and taking logs gives (A.7). For the elasticities, $\log r = \frac{1}{\rho}\log Y + \frac{1}{\mu}\log y$, so each revenue elasticity is $1/\mu$ times the corresponding output elasticity plus the direct $\log Y$ term:

$$\frac{\partial \log r}{\partial \log \phi_h} = \frac{\beta}{\kappa_f \mu} = \frac{1}{\mu-\kappa_f}, \qquad \frac{\partial \log r}{\partial \log \omega} = -\frac{\beta}{\mu}\,\frac{\partial \log q}{\partial \log \omega} = -\frac{\kappa_f(1-\tilde{x})}{\mu-\kappa_f},$$
$$\frac{\partial \log r}{\partial \log c_k} = -\frac{\beta}{\mu}\,\frac{\partial \log q}{\partial \log c_k} = -\frac{\kappa_f \tilde{x}}{\mu-\kappa_f},$$

using Shephard's lemma (A.6), and

$$\frac{\partial \log r}{\partial \log Y} = \frac{1}{\rho} + \frac{1}{\mu}\cdot\frac{\beta}{\rho} = \frac{1}{\rho}\left(1 + \frac{\kappa_f}{\mu-\kappa_f}\right) = \frac{\mu}{\rho\,(\mu-\kappa_f)} = \frac{\mu-1}{\mu-\kappa_f} = \varrho,$$

the last equality because $\mu/\rho = 1/(\rho-1) = \mu - 1$.

(iii) Taking logs of $\omega\, I^d = (\kappa_f/\mu)(1-\tilde{x})\, r$,

$$\log I^d = \log\frac{\kappa_f}{\mu} + \log(1-\tilde{x}) + \log r - \log\omega.$$

Differentiating with respect to $\log\phi_h$ and $\log Y$, only r moves, giving $1/(\mu-\kappa_f)$ and ϱ. With respect to $\log\omega$, using (A.2) for the threshold term and (A.8) for the revenue term,

$$\frac{\partial \log I^d}{\partial \log \omega} = -\,h(\tilde{x}) - \frac{\kappa_f(1-\tilde{x})}{\mu-\kappa_f} - 1 = -\,\mathcal{F}(\tilde{x}),$$

each term strictly negative. With respect to $\log c_k$, the threshold term is $+h(\tilde{x})$ by (A.2), so

$$\frac{\partial \log I^d}{\partial \log c_k} = h(\tilde{x}) - \frac{\kappa_f\,\tilde{x}}{\mu-\kappa_f} = -\,g(\tilde{x}),$$

which proves (27). The identity (A.11) follows because the displacement terms

cancel: $\mathcal{F} + g = 1 + \kappa_f/(\mu - \kappa_f) = \mu/(\mu - \kappa_f) = \rho\varrho$.

(iv) The threshold in part (i) is that of Lemma 1, which is (14) at the firm's prices. At the optimal output, marginal cost equals marginal revenue $r/(\mu y)$, and since TC is homogeneous of degree $1/\kappa_f$ in y, the minimised cost at these prices is $TC = \kappa_f\, y\, TC'(y) = (\kappa_f/\mu)\, r$; by part (i) expenditure is equal across tasks, so the capital bill is the share $\tilde{x}$ of it and the labour bill the share $1 - \tilde{x}$, which are (10). Equating marginal revenue to marginal cost delivers MRPL $= \omega$ at the chosen employment. Conversely the equilibrium allocation satisfies exactly these conditions at (ω_i, ϕ_{ih}); since the solution at given (ω, ϕ_h) is unique (strict concavity in y; the unique threshold of Lemma 1), the two coincide.

(v) Each wage on the perceived supply curve corresponds to exactly one employment level, because $l^s(w)$ is continuous and strictly increasing ($\varepsilon \geq \eta > 0$ everywhere), so the firm's choice can be described by its employment l instead of its wage, with $w^s(l)$ well defined and strictly increasing; profit is then strictly concave in l, which implies quasi-concavity in the wage. Write profit as $S(l) - C(l)$, where $S(l)$ is the maximised revenue net of capital costs at employment l and $C(l) \equiv w^s(l)\, l$. By the envelope theorem, $S'(l) = \text{MRPL}(l)$; by part (iv) and strict monotonicity of l^d in ω from (iii), $S'(l) = \omega^d(l)$, the inverse of $l^d(\cdot\,; \phi_h)$, strictly decreasing in l. On the cost side, $C'(l) = w^s(l)\big(1 + 1/\varepsilon(l)\big) = w^s(l)/\delta(l)$: along the supply curve the share, and hence $1/\delta$, is non-decreasing in l, and $w^s(l)$ is strictly increasing, so $C'(l)$ is strictly increasing. Therefore $S'(l) - C'(l)$ is strictly decreasing, so profit is strictly concave in l and hence strictly quasi-concave in the posted wage, and the wage at which MRPL $= w/\delta$ is the unique global maximiser. □

The lemmas at the extensive margin. With the positive intercept $R(0) > 0$ of Section 4.2, the lemmas carry over as follows.

- *Task assignment.* The argument of Lemma 1(i) goes through unchanged except at the bottom of the task space: capital is strictly cheaper on $[0, x)$

with $x = R^{-1}(\omega/c)$ when $\omega/c > R(0)$, and on no task when $\omega/c \leq R(0)$, in which case $x = 0$. At firm i's prices $(c_{k,i}, \omega_i)$ this is the adoption rule (31).

- *The threshold.* The threshold is non-decreasing in ω/c and strictly increasing on the interior branch, where (A.2) applies as stated. Strictly below the cutoff, $\omega/c < R(0)$, it is locally constant, so h and g are replaced by zero there.
- *At the cutoff.* At $\omega/c = R(0)$ the threshold is zero but need not be locally constant, since any rise in ω/c moves it onto the interior branch; the proof of Proposition 5 never differentiates the threshold there.
- *Costs and demands.* In Lemma 2, the unit cost (A.4) at the corner is the all-labour cost, $\log q = \int_0^1 \log(\omega/\phi_I(Mu))\,du$. Shephard's lemma (A.6) holds on both branches with $\tilde{x} = 0$ at the corner, since $1 - \tilde{x}$ is continuous across the cutoff, so the elasticities (A.9) and the identity (A.11) hold on both branches away from the cutoff. At the cutoff, $\tilde{x}$ and q, and hence revenue and labour demand, are continuous in ω, so labour demand is strictly decreasing in ω across the cutoff as well.
- *Non-adopters.* A non-adopter has $k_i = 0$ and $\alpha_i = 0$; the reduced form (11) and the aggregation of Section 2.3 hold with the convention $0^0 = 1$.
- *Global optimality.* Lemma 2(v) carries over. Labour demand is continuous and strictly decreasing in ω on both branches and across the cutoff, so its inverse $\omega^d(I)$, and hence $S'(I)$, is continuous and strictly decreasing in I, with a kink at the employment level of the cutoff. Profit therefore remains strictly concave in employment, and the wage at which MRPL $= w/\delta$ is the unique global maximiser.

A.2 Proof of Proposition 1

Regularity conditions. Maintain the assumptions of the model and the integrability condition (A.3). When physical capital is present, also assume $\mu - 1 \geq \kappa_p/(1+e)$.

Proof. I first establish that each proposed aggregate output determines a unique allocation satisfying firm optimality and labour-market clearing. I then show that exactly one proposed output also clears the goods and physical-capital markets.

Fix $Y > 0$. When physical capital is present, assign the candidate price

$$p_p(Y) = \left(\frac{\kappa_p}{\mu} Y\right)^{1/(1+e)}. \tag{A.12}$$

This price will clear its market if goods-market clearing holds. Automation capital is available at its competitive price c_k.

Optimising inputs and automation gives

$$r_i = Y^{\varrho_p}\, \Xi_i(\omega_i), \qquad I_i^d = Y^{\varrho_p}\, \Xi_i^I(\omega_i), \tag{A.13}$$

where

$$\varrho_p = \begin{cases} \dfrac{\mu - 1}{\mu - \kappa_f}, & \kappa_p = 0, \\ \dfrac{\mu - 1 - \kappa_p/(1+e)}{\mu - \kappa}, & \kappa_p > 0, \end{cases}$$

so that ϱ_p equals ϱ of Lemma 2 without physical capital, which is also the exponent $1/[\rho(1 - \kappa_f/\mu)]$ in (19), since $1/\rho = (\mu - 1)/\mu$. The assumptions imply $0 \le \varrho_p < 1$. Writing $\tilde{x}_i(\omega)$ for the firm's automated share at prices $(c_{k,i}, \omega)$ and $\alpha_i(\omega) \equiv (\kappa_f/\mu)\, \tilde{x}_i(\omega)$,

$$\begin{aligned} \Xi_i(\omega) &\equiv \left[\tilde{\phi}_i\big(\tilde{x}_i(\omega)\big)^{\kappa_f/\mu} \left(\frac{\alpha_i(\omega)}{c_{k,i}}\right)^{\alpha_i(\omega)} \left(\frac{\kappa_f/\mu - \alpha_i(\omega)}{\omega}\right)^{\kappa_f/\mu - \alpha_i(\omega)} \left(\frac{\kappa_p}{\mu}\right)^{\frac{\kappa_p}{\mu}\frac{e}{1+e}}\right]^{1/(1-\kappa/\mu)}, \\ \Xi_i^I(\omega) &\equiv \frac{\kappa_f}{\mu}\big(1 - \tilde{x}_i(\omega)\big)\, \frac{\Xi_i(\omega)}{\omega}, \end{aligned} \tag{A.14}$$

with $\kappa = \kappa_f + \kappa_p$: revenue and labour demand at unit aggregate output, with physical capital priced at (A.12). Without physical capital the last factor is one and Ξ_i is the Ξ_i of (19); the second line is Lemma 2(iii). The cost-and-demand characterisation

of Lemma 2, with firm-specific installation costs and the physical-capital input included, gives positive, continuous, strictly decreasing functions Ξ_i, Ξ_i^I, with

$$\Xi_i^I(0+) = \infty, \qquad \Xi_i^I(\infty) = 0. \tag{A.15}$$

The second limit holds because Ξ_i is decreasing: for $\omega \geq \omega_0$, $\Xi_i^I(\omega) \leq (\kappa_f/\mu)\, \Xi_i(\omega_0)/\omega$. These properties hold with non-adoption (Appendix A.1).

Conditional labour-market allocation. Write $s_i \equiv s_{i|j}$. Worker sorting (1) and wage setting imply

$$w_i = \Phi_j^{1/\varsigma}\, (s_i/T_i)^{1/\varsigma}, \qquad \omega_i = \Phi_j^{1/\varsigma}\, f_i(s_i), \qquad f_i(s) \equiv \frac{(s/T_i)^{1/\varsigma}}{\delta(s)}. \tag{A.16}$$

Since δ is positive and decreasing, f_i is continuous and strictly increasing.

First fix a market wage index $\Phi_j > 0$. Let $\Theta_j(\Phi_j)$ denote market employment divided by Y^{ϱ_P}. Firm labour-market clearing requires

$$\Xi_i^I\big(\Phi_j^{1/\varsigma} f_i(s_i)\big) = \Theta_j(\Phi_j)\, s_i, \qquad \sum_{i \in j} s_i = 1. \tag{A.17}$$

For each firm, $\Xi_i^I(\Phi_j^{1/\varsigma} f_i(s))/s$ decreases continuously from infinity as s increases from zero. For $\Theta \geq \max_{i \in j} \Xi_i^I(\Phi_j^{1/\varsigma} f_i(1))$, the individual equations determine unique shares. At the lower bound, the firm attaining the maximum has share one and every other firm a positive share, so their sum is at least one; it decreases continuously to zero as $\Theta \to \infty$. Thus (A.17) determines a unique $\Theta_j(\Phi_j)$ and share vector.

Increasing Φ_j reduces every share at a fixed Θ, so $\Theta_j(\Phi_j)$ strictly decreases. Continuity, (A.15) and the share normalisation imply

$$\Theta_j(0+) = \infty, \qquad \Theta_j(\infty) = 0. \tag{A.18}$$

I now determine the market wage indices. By (1), market employment must equal both firms' demand and workers' supply, $Y^{\varrho_p}\Theta_j(\Phi_j) = \mathcal{L}\,\Phi_j^{\eta/\varsigma}/\sum_{j'}\Phi_{j'}^{\eta/\varsigma}$. Equivalently,

$$\Theta_j(\Phi_j) = \mathfrak{q}\,\Phi_j^{\eta/\varsigma}, \qquad Y^{\varrho_p}\sum_j \Theta_j(\Phi_j) = \mathcal{L} \tag{A.19}$$

for a common $\mathfrak{q} > 0$. Since $\Theta_j(\Phi_j)/\Phi_j^{\eta/\varsigma}$ decreases continuously from infinity to zero, the first equation uniquely determines $\Phi_j(\mathfrak{q})$, decreasing in $\mathfrak{q}$. Hence $\sum_j \Theta_j(\Phi_j(\mathfrak{q}))$ increases continuously from zero to infinity, and the second equation uniquely determines $\mathfrak{q}$.

This establishes a unique, continuous labour-market allocation for every $Y > 0$. The associated wage conditions give global firm optima by the argument of Lemma 2(v): revenue net of capital costs is strictly concave in employment and wage expenditure is strictly convex.

Aggregate consistency. An increase in Y weakly raises Y^{ϱ_p}, since $\varrho_p \geq 0$ (this is where $\mu - 1 \geq \kappa_p/(1+e)$ is used). The second equation of (A.19) then requires a weakly lower $\sum_j \Theta_j$, so $\mathfrak{q}$ weakly falls and every Φ_j weakly rises. Every ω_i also weakly rises. Otherwise, (A.16) would imply a lower share s_i, making the right side of (A.17) fall while its left side weakly rises, a contradiction. Thus $\omega_i(Y)$ is nondecreasing in Y.

Define normalised excess revenue by

$$\mathcal{Z}(Y) \equiv \frac{\sum_i r_i(Y) - Y}{Y} = Y^{\varrho_p - 1}\sum_i \Xi_i\big(\omega_i(Y)\big) - 1. \tag{A.20}$$

Because $\varrho_p < 1$ and each $\Xi_i(\omega_i(Y))$ is positive and nonincreasing, $\mathcal{Z}$ is continuous and strictly decreasing. Bounding the sum by its value at $Y = 1$, on either side of one, gives

$$\lim_{Y\to 0}\mathcal{Z}(Y) = \infty, \qquad \lim_{Y\to\infty}\mathcal{Z}(Y) = -1.$$

The intermediate value theorem therefore gives exactly one $Y^* > 0$ satisfying

$\mathcal{Z}(Y^*) = 0$.

Goods-market clearing holds at Y^*. Physical-capital clearing follows because, along the candidate prices (A.12),

$$\frac{K^{p,d}(Y)}{K^{p,s}(Y)} = \frac{(\kappa_p/\mu)\sum_i r_i(Y)}{p_p(Y)^{1+e}} = 1 + \mathcal{Z}(Y).$$

Automation supply meets firms' demand at its fixed competitive price. All remaining quantities follow uniquely. Every equilibrium must satisfy the equations above, so the constructed equilibrium exists and is unique. □

A.3 Proof of Proposition 2

Proof. Both firms are in the one market with the same amenity T, so by (1) the posted wage is $w_i = (s_i \Phi_j / T)^{1/\varsigma}$, which rises with the within-market share. By (13) and (A.1), the markdown $\delta_i = \delta(s_i)$ falls with it.

Step 1: the share ranking. Suppose $s_H \leq s_L$. Then $w_H \leq w_L$ and $\delta_H \geq \delta_L$, so $\omega_H = w_H/\delta_H \leq \omega_L$. By Lemma 2(iii)–(iv), labour demand at the common Y is strictly increasing in ϕ_h and strictly decreasing in ω, so $l_H = l^d(\omega_H; \phi_{Hh}) > l^d(\omega_L; \phi_{Lh}) = l_L$. But $l_i = s_i L$ in the single market, so $s_H \leq s_L$ gives $l_H \leq l_L$: a contradiction. Hence $s_H > s_L$ and $l_H > l_L$.

Step 2: labour market power. With $s_H > s_L$, $w_H > w_L$ and $\delta_H < \delta_L$, hence $\omega_H > \omega_L$. This proves part (ii).

Step 3: automation and revenue. Both firms buy automation capital at the price c_k and $\omega_H > \omega_L$, so Lemma 1(ii) gives $\tilde{x}_H > \tilde{x}_L$. Finally, by (10),

$$r_i = \frac{\omega_i l_i}{(\kappa_f/\mu)(1 - \tilde{x}_i)}.$$

The numerator is strictly larger for firm H and the denominator is smaller, so $r_H > r_L$. This proves parts (i) and (iii).

Step 4: profit margins. By (17), $\tilde{\pi}_i/r_i = (1 - \kappa_f/\mu) + (1 - \delta_i)(\kappa_f/\mu)(1 - \tilde{x}_i)$. The first

term is common to both firms, so $\tilde{\pi}_H/r_H > \tilde{\pi}_L/r_L$ if and only if (23) holds. This proves part (iv). □

A.4 Comparative statics

Maintained assumption.

(A2) In the analytical economy, the equilibria along the family (25) exist for all sufficiently small $\theta \geq 0$ and are right-differentiable in θ at $\theta = 0$, and the oligopsonistic equilibrium is differentiable in c_k.

The total differentials. In equilibrium $l_i = l^d(\omega_i, c_k, Y; \phi_{ih})$ by Lemma 2(iv). Along any path on which ϕ_{ih} is fixed, write $z \equiv -d\log c_k$ for the fall in the automation price and $\mathcal{F}_i \equiv \mathcal{F}(\tilde{x}_i)$. Totally differentiating with the four elasticities (A.9),

$$d\log l_i = g_i\, z \;-\; \mathcal{F}_i\, d\log \omega_i \;+\; \varrho\, d\log Y: \tag{A.21}$$

$g_i z$ is the direct labour-demand shift from cheaper automation at a fixed marginal labour cost, $\mathcal{F}_i$ the absolute elasticity of labour demand with respect to that cost, and $\varrho\, d\log Y$ the common demand-expansion term. The firm's automation depends on the relative factor cost ω_i/c_k (Lemma 1), so write $a_i \equiv d\log\omega_i + z$ for its change. Using (A.11), (A.21) becomes

$$d\log l_i = \rho\varrho\, z \;-\; \mathcal{F}_i\, a_i \;+\; \varrho\, d\log Y, \tag{A.22}$$

in which only the middle term differs across firms. Likewise, by (A.8),

$$d\log r_i = \varrho\, d\log Y + \frac{\kappa_f}{\mu - \kappa_f}\big[z - (1-\tilde{x}_i)\, a_i\big], \tag{A.23}$$

in which, by Shephard's lemma (A.6), the induced movement of the threshold has no first-order effect.

Two equilibrium identities. Propositions 3 and 4 are comparative statics of the analytical economy along a smooth path of equilibria ((A2)): in the wedge parameter θ at $\theta = 0$, or in the automation price c_k. All differentials are evaluated at the initial equilibrium, with ϕ_{ih} and T fixed. With a single market, $\Phi = \Gamma^{\eta}\Phi_j^{\eta/\varsigma}$, so $s_j = 1$ identically: $l_i = s_i L$, $d\log l_i = d\log s_i$, and $ds_H + ds_L = 0$. Write $ds \equiv ds_H = -ds_L$ and $\varpi_i \equiv r_i/Y$ for the revenue weights, and

$$\Delta_Y \;\equiv\; g_H - g_L \;=\; \mathcal{F}_L - \mathcal{F}_H, \tag{A.24}$$

by (A.11). Incidence is concentrating exactly when $\Delta_Y > 0$ and equalising exactly when $\Delta_Y < 0$.

Lemma 3 (Two equilibrium identities)**.** *Along either path,*

$$\frac{ds}{s_H s_L} \;=\; \mathcal{F}_L\, a_L - \mathcal{F}_H\, a_H, \tag{A.25}$$

$$A_H\, a_H + A_L\, a_L \;=\; \varrho\Big(\rho + \frac{\kappa_f}{1-\kappa_f}\Big) z, \qquad A_i \equiv s_i\,\mathcal{F}_i + \frac{\kappa_f\,\varrho}{1-\kappa_f}\,\varpi_i\,(1-\tilde{x}_i) \;>\; 0. \tag{A.26}$$

The first identity says that workers move towards the firm whose marginal labour cost rises by less relative to the price of automation capital, each firm's change weighted by its labour-demand elasticity. The second combines the two market-clearing conditions: a fall in the automation price raises the marginal labour cost relative to the price of automation capital on average across firms, with weights that add each firm's employment share to its share of labour cost.

Proof. For (A.25), subtract (A.22) for L from that for H: the common terms cancel, and $d\log s_H - d\log s_L = ds/s_H + ds/s_L = ds/(s_H s_L)$.

For (A.26), labour clearing gives $s_H\, d\log s_H + s_L\, d\log s_L = ds_H + ds_L = 0$. Weighting (A.22) by s_i and summing,

$$\varrho\; d\log Y = \sum_i s_i\,\mathcal{F}_i\, a_i - \rho\varrho\; z.$$

Goods clearing, Definition 1(iii), gives $d\log Y = \sum_i \varpi_i \, d\log r_i$; substituting (A.23), using $\sum_i \varpi_i = 1$ and $1-\varrho = (1-\kappa_f)/(\mu-\kappa_f)$,

$$d\log Y = \frac{\kappa_f}{1-\kappa_f}\Big[z - \sum_i \varpi_i \,(1-\tilde{x}_i)\, a_i\Big].$$

Multiplying by ϱ and equating the two expressions for $\varrho \, d\log Y$ gives (A.26). Each A_i is a sum of products of strictly positive terms. □

A.5 Proof of Proposition 3

The family (25) nests the two equilibria. At every θ, the firm's output, threshold and factor demands are given by Lemma 2 at (ω_i, ϕ_{ih}), employment satisfies $l^d(\omega_i;\phi_{ih}) = s_i L$, workers sort on posted wages by (1), and the goods market clears, Definition 1(iii); only the link between the posted wage and the marginal labour cost changes with θ. At $\theta = 1$ the rule is the wage-posting optimum (7)–(12), so the family ends at the oligopsonistic equilibrium. All differentials below are evaluated at $\theta = 0$, with the automation price fixed, so $z = 0$ and $a_i = d\log\omega_i$.

Proof. Step 1: the ranking at the benchmark. At $\theta = 0$, $\omega_i = w_i = (s_i\Phi_j/T)^{1/\varsigma}$. Suppose $s_H \le s_L$. Then $\omega_H \le \omega_L$, and Lemma 2(iii)–(iv) gives $l_H > l_L$, contradicting $l_i = s_i L$. Hence $s_H > s_L$, and since $\delta(\cdot)$ is strictly decreasing,

$$\Delta\mathfrak{d} \;\equiv\; \delta(s_L) - \delta(s_H) \;>\; 0.$$

Step 2: marginal labour costs and shares. From (25), $\log\omega_i = \log w_i - \log\{1 - \theta\,[1-\delta(s_i)]\}$. At $\theta = 0$ the differential of the last term is $[1-\delta(s_i)]\,d\theta$; the term in ds_i carries the factor θ and vanishes. Sorting gives $d\log w_i = d\nu + \frac{1}{\varsigma}\,d\log s_i$ with $d\nu \equiv \frac{1}{\varsigma}\,d\log\Phi_j$ common to both firms. Subtracting,

$$a_H - a_L \;=\; \frac{ds}{\varsigma\, s_H s_L} + \Delta\mathfrak{d}\, d\theta. \tag{A.27}$$

Now suppose $a_H \leq 0$. With $z = 0$, (A.26) reads $A_H a_H + A_L a_L = 0$, so $a_L \geq 0$, and (A.25) then gives $ds \geq 0$. The left side of (A.27) is then nonpositive while the right side is strictly positive: a contradiction. Hence $a_H > 0$, so $a_L < 0$ by (A.26), and $ds < 0$ by (A.25). This proves part (ii).

Step 3: wages and thresholds. The wage premium moves with the shares, $d \log w_H - d \log w_L = ds/(\varsigma s_H s_L) < 0$, which with $ds < 0$ proves part (i). By (A.2) at fixed c_k, $d\tilde{x}_i = (1 - \tilde{x}_i)\, h_i\, a_i$, so $d\tilde{x}_H > 0 > d\tilde{x}_L$ and the gap in automated shares widens, part (iii). The second form of (A.2) gives $d \log(1 - \tilde{x}_i) = -h_i\, a_i$; subtracting the two firms and using $a_H > 0 > a_L$ gives (26), part (iv). □

A.6 Proof of Proposition 4

Here the path is in c_k with the wage-posting rule (12), $\omega_i = w_i/\delta(s_i)$, and $z > 0$.

Proof. Step 1: both relative factor costs rise. Sorting gives $d \log w_i = dv + \frac{1}{\varsigma}\, d \log s_i$, and the wage rule adds the markdown feedback, $d \log \omega_i = d \log w_i - [\delta'(s_i)/\delta_i]\, ds_i$. With $ds_L = -ds$,

$$a_H - a_L \;=\; \mathfrak{c}_s\, ds, \qquad \mathfrak{c}_s \;\equiv\; \frac{1}{\varsigma\, s_H s_L} - \frac{\delta'(s_H)}{\delta_H} - \frac{\delta'(s_L)}{\delta_L} \;>\; 0, \tag{A.28}$$

by (A.1). Substituting $ds = (a_H - a_L)/\mathfrak{c}_s$ into (A.25) and collecting,

$$\left(1 + \mathfrak{c}_s s_H s_L\, \mathcal{F}_H\right) a_H \;=\; \left(1 + \mathfrak{c}_s s_H s_L\, \mathcal{F}_L\right) a_L.$$

Both coefficients are positive, so a_H and a_L have the same sign. If both were nonpositive, the left side of (A.26) would be nonpositive while the right side is strictly positive for $z > 0$. Hence $a_H > 0$ and $a_L > 0$, and by (A.2) $d\tilde{x}_i = (1 - \tilde{x}_i)\, h_i\, a_i > 0$ at both firms. This proves part (i).

Step 2: reallocation. Substituting $a_H = a_L + \mathfrak{c}_s\, ds$ into (A.25),

$$\left(\frac{1}{s_H s_L} + \mathfrak{c}_s\, \mathcal{F}_H\right) ds \;=\; (\mathcal{F}_L - \mathcal{F}_H)\, a_L \;=\; \Delta_Y\, a_L .$$

The coefficient on the left and a_L are positive, so $\operatorname{sign}(ds) = \operatorname{sign}(\Delta_Y)$, and $ds = 0$ exactly when $\Delta_Y = 0$. The concentrating and equalising cases follow from the definitions after (A.24); one firm gains only if the other loses, since $ds_L = -ds$. This proves part (ii).

Step 3: markdowns. Since $\delta_i = \delta(s_i)$ and $ds_L = -ds$,

$$d\delta_H = \delta'(s_H)\, ds, \qquad d\delta_L = -\,\delta'(s_L)\, ds, \qquad d(\delta_L - \delta_H) = \left[|\delta'(s_L)| + |\delta'(s_H)|\right] ds,$$

with $\delta' < 0$ by (A.1). Thus $ds > 0$ lowers δ_H, raises δ_L, and widens the gap; all three movements reverse when $ds < 0$. Part (ii) supplies the incidence statements. This proves part (iii). □

A.7 Proof of Proposition 5

The adoption ordering. The statement in Section 4.1 that firms with identical installation requirements retain the ranking of Proposition 2 follows from the adoption rule. Take two firms i and i' in the same market with the same amenity and $\delta_i < \delta_{i'}$. Since $\delta(\cdot)$ is strictly decreasing, $s_{i|j} > s_{i'|j}$. The posted wage rises with the share, $w_i = (s_{i|j}\Phi_j/T)^{1/\varsigma}$, and the markdown falls with it, so $\omega_i = w_i/\delta_i > \omega_{i'}$. If both firms draw the same requirement b and firm i' adopts, then $\omega_i > \omega_{i'} > c_k\, b R(0)$, so firm i adopts, and by the interior branch of (31) $\tilde{x}_i > \tilde{x}_{i'}$.

Setup. Work in the analytical economy with $\phi_{Hh} > \phi_{Lh}$, a common amenity T, fixed total employment, no physical capital, a common installation requirement b, and the increasing task schedule of Assumption 1 with the intercept $R(0) > 0$. Wage-setting power is introduced along the family (25), with c_k, b, productivities,

the amenity and the task schedule fixed. Assumption (A2) is used here only in a weaker form: equilibria exist for all sufficiently small $\theta \geq 0$ and converge to the equilibrium without labour markdowns as $\theta \to 0$.

Proof. Step 1: neither firm adopts at $\theta = 0$. Define the common adoption threshold $\bar{\omega} \equiv c_k b R(0)$; by hypothesis $\omega_H(0) = \bar{\omega}$. At $\theta = 0$, $\omega_i = w_i = (s_i \Phi_j / T)^{1/\varsigma}$. Suppose $s_H \leq s_L$. Then $\omega_H \leq \omega_L$, and Lemma 2(iii)–(iv), which hold at the corner by Appendix A.1, give $l_H > l_L$, contradicting $l_i = s_i L$. Hence $s_H(0) > s_L(0)$ and $\omega_L(0) < \omega_H(0) = \bar{\omega}$. The adoption rule (31) assigns no automated tasks at equality, so $d_H(0) = d_L(0) = 0$. Since $\delta(\cdot)$ is strictly decreasing,

$$\Delta\mathfrak{d} \equiv \delta(s_L(0)) - \delta(s_H(0)) > 0.$$

Step 2: the economy without automation. Consider the auxiliary economy in which both firms must use labour on every task, so $\tilde{x}_i \equiv 0$. It coincides with the actual economy at $\theta = 0$. Wide hats denote its allocations and narrow hats their derivatives and weights at $\theta = 0$: $\hat{a}_i \equiv d \log \widehat{\omega}_i / d\theta$ and $\hat{a}_s \equiv d\widehat{s}_H / d\theta$, and all weights and shares are evaluated at $\theta = 0$. With $\tilde{x}_i = 0$ and $h_i = g_i = 0$, (A.10) gives the same labour-demand elasticity at both firms,

$$\mathcal{F}_0 \equiv \frac{\mu}{\mu - \kappa_f} > 0,$$

and the two identities of Lemma 3, whose derivation uses only labour demand, revenue and the two clearing conditions, hold with $z = 0$:

$$\frac{\hat{a}_s}{s_H s_L} = \mathcal{F}_0 (\hat{a}_L - \hat{a}_H), \qquad \hat{A}_H \hat{a}_H + \hat{A}_L \hat{a}_L = 0, \qquad \hat{A}_i = s_i \mathcal{F}_0 + \frac{\kappa_f \varrho}{1 - \kappa_f} \varpi_i > 0. \tag{A.29}$$

Step 3: wage-setting power raises H's marginal labour cost in the auxiliary economy. As in Step 2 of the proof of Proposition 3, the wage rule and sorting give (A.27), $\hat{a}_H - \hat{a}_L = \hat{a}_s / (\varsigma s_H s_L) + \Delta\mathfrak{d}$; the change in $\delta(s_i)$ inside (25) is multiplied by θ

and has no first-order effect at $\theta = 0$. Substituting the first identity in (A.29),

$$\left(1 + \frac{\mathcal{F}_0}{\varsigma}\right)(\hat{a}_H - \hat{a}_L) \;=\; \Delta\mathfrak{d} \;>\; 0,$$

and with the second identity,

$$\hat{a}_H \;=\; \frac{\hat{A}_L}{\hat{A}_H + \hat{A}_L}\,\frac{\Delta\mathfrak{d}}{1 + \mathcal{F}_0/\varsigma} \;>\; 0, \qquad \hat{a}_L \;=\; -\,\frac{\hat{A}_H}{\hat{A}_H + \hat{A}_L}\,\frac{\Delta\mathfrak{d}}{1 + \mathcal{F}_0/\varsigma} \;<\; 0. \tag{A.30}$$

The same equations give local uniqueness and smoothness of the auxiliary equilibrium. Set $\Delta\mathfrak{d}$ to zero: the first identity and (A.27) then give $\hat{a}_H = \hat{a}_L$, the second gives $\hat{a}_H = \hat{a}_L = 0$, the first gives $\hat{a}_s = 0$, and goods-market clearing gives $d \log \widehat{Y} = 0$. The Jacobian of the auxiliary equilibrium system is therefore nonsingular. Its equations are smooth, since no threshold moves, so the implicit function theorem gives a unique differentiable auxiliary equilibrium near $\theta = 0$, which justifies (A.30). Hence, for every sufficiently small $\theta > 0$,

$$\widehat{\omega}_H(\theta) \;>\; \widehat{\omega}_H(0) \;=\; \bar{\omega}. \tag{A.31}$$

Step 4: continued non-adoption is not an equilibrium. In the actual economy, firm L starts strictly below the threshold, so by continuity $\omega_L(\theta) < \bar{\omega}$ and $d_L(\theta) = 0$ for all sufficiently small $\theta > 0$. Suppose firm H also did not adopt at such a θ. The actual equilibrium would then solve the equations of the auxiliary economy and, by their local uniqueness, coincide with the auxiliary equilibrium. By (A.31), $\omega_H(\theta)/(c_k b) > R(0)$. Since the task schedule is continuous and strictly increasing, automation capital is then strictly cheaper than labour on a positive interval of tasks, and (31) requires $\tilde{x}_H(\theta) > 0$, a contradiction. Hence

$$d_H(0) = d_L(0) = 0, \qquad d_H(\theta) = 1, \quad d_L(\theta) = 0 \quad \text{for all sufficiently small } \theta > 0. \;\square$$

A.8 Proof of Proposition 6

Maintained identification assumptions. The distributional assumptions and normalisations of Section 5.3 hold, and the externally calibrated parameters of Section 5.4 are known. Aggregate prices and labour-market indices are held fixed. The number of firms in each market is taken as given, with a positive share of single-firm markets. Parameter values are interior, with $\sigma_\phi > 0$, $\upsilon > 0$ and $|\mathcal{R}_{\phi T}| < 1$, and wages, employment, revenue and market membership are observed without error.

Proof. Write $u_i = \tilde{x}_i$ and $\mathrm{t}_i = \log T_i$. Expectations are across firms, and aggregate equilibrium objects are finite, positive and constant across observations.

Step 1: recover amenities and automation intensity. Labour supply (1), the markdown schedule (13) and labour demand (S1.3) imply

$$\mathrm{t}_i = \log s_{i|j} + \frac{\varsigma}{\eta}\log s_j - \varsigma \log w_i + \mathfrak{t}_T,$$
$$\omega_i = \frac{w_i}{\delta(s_{i|j})}, \qquad u_i = 1 - \frac{\mu}{\kappa_f}\frac{\omega_i l_i}{r_i}.$$

The normalisation $\mathbb{E}[\mathrm{t}_i] = 0$ fixes the common constant $\mathfrak{t}_T$. The Pareto restriction then identifies

$$\iota = \mathrm{Var}(\mathrm{t}_i)^{-1/2}, \qquad z_i = z(\mathrm{t}_i) = F_N^{-1}\big(F_T(e^{\mathrm{t}_i})\big),$$

where F_T and F_N are the amenity and standard normal distribution functions. Thus $(\mathrm{t}_i, z_i, \omega_i, u_i)$ are recoverable from observables.

Step 2: identify the task schedule. Restrict attention to single-firm markets. There the within-market share is one, so the markdown is the constant $\delta(1)$ and the firm's employment depends only on its own marginal labour cost and amenity. Because firm draws are i.i.d. and the number of firms in each market is given, firms in single-firm markets are a random sample from the same primitive distributions

as all firms, so parameters identified from them are the population parameters. Define the known constants

$$\mathfrak{c}_b = \mu - \kappa_p + \eta(\mu - \kappa), \qquad \mathfrak{c}_T = \frac{\eta(\mu - \kappa)}{\varsigma} > 0,$$

where $\mu - \kappa > 0$. In these markets, labour supply gives $\log l_i = \eta \log \omega_i + (\eta/\varsigma)\mathfrak{t}_i + \mathfrak{k}_l$. Combining this with production, factor demands and the automation condition (31) yields $\mathcal{H}(u_i) = Z_i$ for adopters, where

$$\mathcal{H}(u) = -\big[\mu - \kappa + (\mathfrak{c}_b - \kappa_f)a\big]\log(1-u) + (\kappa_f a + \mathfrak{c}_b\beta_R)\,u - \frac{\kappa_f\beta_R}{2}\,u^2,$$
$$Z_i = \log\phi_{ih} - \mathfrak{c}_b \log b_i - \mathfrak{c}_T\mathfrak{t}_i - \mathfrak{c}_b \log c_k - \mathfrak{k}_0.$$

Here $\mathfrak{k}_l$ and $\mathfrak{k}_0$ are common constants. For non-adopters, $Z_i = \mathfrak{c}_b \log[\omega_i/(c_k b_i)] \le 0$. Moreover, $\mathcal{H}(0) = 0$, $\mathcal{H}(u) \to \infty$ as $u \to 1$, and

$$\mathcal{H}'(u) = (\mathfrak{c}_b - \kappa_f u)\left(\beta_R + \frac{a}{1-u}\right) + \frac{\mu - \kappa}{1-u} > 0,$$

because $\mathfrak{c}_b - \kappa_f = (1+\eta)(\mu - \kappa) > 0$ and $\beta_R > -a$. Hence $u_i = 0$ when $Z_i \le 0$, and $u_i = \mathcal{H}^{-1}(Z_i)$ otherwise.

The Gaussian copula (38) and independent lognormal installation costs imply

$$\log\phi_{ih} = \mu_\phi + \lambda z_i + \epsilon_{\phi i}, \qquad \log b_i = -\frac{\upsilon^2}{2} + \epsilon_{bi}, \qquad \lambda = \sigma_\phi \mathcal{R}_{\phi T},$$

where the residuals are independent centred normals, independent of amenities, with variances $\sigma_\phi^2(1 - \mathcal{R}_{\phi T}^2)$ and υ^2. Consequently,

$$Z_i = m + \lambda z_i - \mathfrak{c}_T\mathfrak{t}_i + \epsilon_{\phi i} - \mathfrak{c}_b\epsilon_{bi}, \qquad \sigma_Z^2 \equiv \mathrm{Var}(Z_i \mid \mathfrak{t}_i) = \sigma_\phi^2(1 - \mathcal{R}_{\phi T}^2) + \mathfrak{c}_b^2\upsilon^2,$$

for a common intercept m. The observable conditional distribution function

therefore satisfies

$$F_N^{-1}\big[\Pr(u_i \le u \mid \mathfrak{t}_i = \mathfrak{t})\big] = \frac{\mathcal{H}(u) - m - \lambda z(\mathfrak{t}) + \mathfrak{c}_T \mathfrak{t}}{\sigma_Z}, \qquad 0 \le u < 1. \tag{A.32}$$

At $u = 0$, (A.32) identifies the coefficients on 1, $z(\mathfrak{t})$ and $\mathfrak{t}$. These functions are linearly independent because $z(\mathfrak{t}) \to -\infty$ at the finite lower endpoint $\mathfrak{t} = -1/\iota$. Since $\mathfrak{c}_T > 0$ is known, the coefficient $\mathfrak{c}_T/\sigma_Z$ identifies σ_Z^2; the other coefficients identify m and λ.

Differencing (A.32) between $u > 0$ and $u = 0$ then identifies $\mathcal{H}(u)$. Linear independence of $\log(1-u)$, u and u^2 identifies β_R from its quadratic coefficient and a from its logarithmic coefficient. This includes $\beta_R = 0$.

Step 3: separate installation costs from productivity heterogeneity. For adopters, the quantity, observable once a and β_R are known,

$$q_i = \log \omega_i - \beta_R u_i + a \log(1-u_i) = \log c_k - \frac{\upsilon^2}{2} + \epsilon_{bi}$$

satisfies, by normal conditioning,

$$\mathbb{E}[q_i \mid u_i = u, \mathfrak{t}_i = \mathfrak{t}] = \log c_k - \frac{\upsilon^2}{2} - \frac{\mathfrak{c}_b \upsilon^2}{\sigma_Z^2}\big[\mathcal{H}(u) - m - \lambda z(\mathfrak{t}) + \mathfrak{c}_T \mathfrak{t}\big], \qquad u > 0.$$

This follows from $\mathrm{Cov}(\epsilon_{bi}, \epsilon_{\phi i} - \mathfrak{c}_b \epsilon_{bi}) = -\mathfrak{c}_b \upsilon^2$. At fixed amenities, automation intensity has positive density throughout $(0,1)$. Variation in $\mathcal{H}(u)$ therefore identifies the slope $-\mathfrak{c}_b\upsilon^2/\sigma_Z^2$, hence υ^2; the level then identifies c_k. Conditioning on automation intensity accounts for selection into adoption.

Finally,

$$\sigma_\phi^2 = \sigma_Z^2 - \mathfrak{c}_b^2 \upsilon^2 + \lambda^2, \qquad \mathcal{R}_{\phi T} = \frac{\lambda}{\sigma_\phi}.$$

All seven parameters are uniquely recovered, so observationally equivalent interior parameter vectors coincide. □

B Solution algorithm

This appendix gives the algorithm that solves the quantitative model of Section 4. The quantitative model extends the baseline model with heterogeneous installation costs, an extensive margin of automation, and physical capital.

The iteration. The equilibrium is computed by iterating on a state vector of employment shares and automation thresholds. The firm-level draws (ϕ_{ih}, T_i, b_i), and hence the firm-specific capital costs $c_{k,i}$, are fixed before the iteration begins. Let $\chi_i \equiv s_{i|j}\, s_j$ denote the economy-wide employment share of firm i — the unconditional choice probability of Supplemental Material S1.1 — so that $I_i = \chi_i\, \mathcal{L}$, and let the state vector be

$$\mathbf{s} \;\equiv\; \big(\{\chi_i\},\, \{\tilde{x}_i\}\big). \tag{B.1}$$

Because the state carries the employment shares directly, labour-market clearing holds at every iteration.

Step 0 (Initialise). Guess $\mathbf{s}^{(0)} = \big(\{\chi_i^{(0)}\},\, \{\tilde{x}_i^{(0)}\}\big)$ with $\chi_i^{(0)} \in (0,1)$, $\sum_{i\in\mathcal{I}} \chi_i^{(0)} = 1$, and $\tilde{x}_i^{(0)} \in [0,1)$ for all i.

Step 1 (Firm-level assignments). Set $I_i = \chi_i^{(n)}\mathcal{L}$ and define the gross-revenue shares

$$\sigma_p \;\equiv\; \frac{\kappa_p}{\mu}, \qquad \alpha_i \;\equiv\; \frac{\kappa_f}{\mu}\tilde{x}_i^{(n)}. \tag{B.2}$$

Let $\lambda_i \equiv \alpha_i + \sigma_p$ be the combined share of the two capital inputs concentrated out below, and let $\nu_i \equiv (\kappa_f/\mu)(1-\tilde{x}_i^{(n)})/(1-\lambda_i)$ be the remaining labour exponent.

Step 2 (Aggregate output and the price of physical capital). In Steps 2–4 the iteration superscript is dropped: $\tilde{x}_i$ denotes the current iterate $\tilde{x}_i^{(n)}$. Solve the goods-market condition

$$Y \;=\; \sum_{i\in\mathcal{I}} \Omega_i^{Q}(\tilde{x}_i; p_p)\, Y^{1/[\rho(1-\lambda_i)]}\, I_i^{\nu_i}, \qquad p_p = \big(\sigma_p\, Y\big)^{1/(1+e)}, \tag{B.3}$$

where

$$\Omega_i^Q(\tilde{x}_i; p_p) \;\equiv\; \left[\tilde{\phi}_i(\tilde{x}_i)^{\kappa_f/\mu}\left(\frac{\alpha_i}{c_{k,i}}\right)^{\alpha_i}\left(\frac{\sigma_p}{p_p}\right)^{\sigma_p}\right]^{1/(1-\lambda_i)}. \tag{B.4}$$

The second equation in (B.3) is the capital-market clearing condition (34). Substituting it into the first leaves one equation in Y, solved by one-dimensional root-finding. For a non-adopter, $\alpha_i = \tilde{x}_i = 0$, and both $(\alpha_i/c_{k,i})^{\alpha_i}$ and $\tilde{\phi}_i(0)$ are evaluated with the convention $0^0 = 1$. The equation follows by substituting the automation-capital and physical-capital demands into gross revenue and then imposing $\sum_i r_i = Y$.

Step 3 (Firm-level objects). Given Y and p_p, compute

$$r_i = \Omega_i^Q(\tilde{x}_i; p_p)\, Y^{1/[\rho(1-\lambda_i)]}\, I_i^{\nu_i}, \qquad r_i^{\mathrm{N}} = \frac{\tilde{\mu}}{\mu} r_i, \qquad k_i = \frac{\alpha_i r_i}{c_{k,i}}, \qquad k_i^p = \frac{\kappa_p r_i}{\mu\, p_p}. \tag{B.5}$$

Step 4 (Wages). Recover within-market shares $s_{i|j} = \chi_i^{(n)} / \sum_{i' \in j} \chi_{i'}^{(n)}$ and the markdown $\delta_i = \delta(s_{i|j})$. The marginal revenue product of labour and the wage are

$$\mathrm{MRPL}_i = \frac{\kappa_f}{\mu}(1-\tilde{x}_i)\frac{r_i}{l_i} = \frac{\kappa_f}{\tilde{\mu}}(1-\tilde{x}_i)\frac{r_i^{\mathrm{N}}}{l_i}, \qquad w_i = \delta_i \mathrm{MRPL}_i.$$

Set $\omega_i = w_i/\delta_i = \mathrm{MRPL}_i$.

Step 5 (Model-implied state). Given the wages $\{w_i\}$, compute the model-implied state $\hat{\mathbf{s}} = \left(\{\hat{\chi}_i\}, \{\hat{\tilde{x}}_i\}\right)$.

Employment shares. Form the wage indices and the implied share from the Fréchet labour supply (1):

$$\Phi_j = \sum_{i' \in j} T_{i'}\, w_{i'}^{\varsigma}, \qquad \Phi = \sum_{j'} \Gamma^{\eta}\, \Phi_{j'}^{\eta/\varsigma}, \qquad \hat{\chi}_i = \frac{T_i\, w_i^{\varsigma}}{\Phi_j} \cdot \frac{\Gamma^{\eta}\, \Phi_j^{\eta/\varsigma}}{\Phi}. \tag{B.6}$$

Automation thresholds. Using ω_i and $c_{k,i}$, the threshold condition (14) gives

the update

$$\hat{\hat{x}}_i \text{ solves } R\big(M\hat{\hat{x}}_i\big) = \frac{\omega_i}{c_{k,i}} \text{ if } \frac{\omega_i}{c_{k,i}} > R(0), \qquad \hat{\hat{x}}_i = 0 \text{ otherwise.} \tag{B.7}$$

Under the power–exponential schedule of Section 5.3, firms with $\omega_i \le c_{k,i}$ are placed at the corner. For adopters, strict monotonicity of R guarantees a unique root in (B.7), which is computed by one-dimensional root-finding.

Step 6 (Dampen and iterate). Update with relaxation weight $\nu \in (0, 1]$:

$$\mathbf{s}^{(n+1)} = \nu\,\hat{\mathbf{s}} + (1-\nu)\,\mathbf{s}^{(n)}. \tag{B.8}$$

If $\|\hat{\mathbf{s}} - \mathbf{s}^{(n)}\|_\infty < 10^{-10}$, stop; otherwise return to Step 1. At convergence, compute firm profits as revenue net of the automation, labour, and physical-capital bills, the physical-capital stock $K^p = \sum_i k_i^p$, the upstream resource cost and profit from (33), owner consumption from (S1.6), and aggregate consumption from (S1.5).

SUPPLEMENTAL MATERIAL – NOT FOR PUBLICATION

LABOUR MARKET POWER, AUTOMATION AND WELFARE

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This Supplemental Material is organised as follows. Section S1 derives the labour supply curve, workers' welfare and social welfare, and the conditions of the model with physical capital. Section S2 extends the characterisation of Section 3 to an economy with two types of local labour market. Section S3 sets out the normalisations. Section S4 describes the data, the measurement of automation capital and the sample, and estimates markdowns from production functions. Section S5 defines the targeted moments, checks local identification at the estimates and explains how the fall in the automation price is computed. Section S6 reports the sensitivity analysis of Section 6.3. Equations, propositions, sections, tables and figures without a prefix refer to the main paper; those with a letter prefix refer to the Online Appendix.

S1 Derivations

S1.1 Section 2: Model

Choice probabilities and the labour supply curve. This derivation follows Appendix A.1 of Azkarate-Askasua and Zerecero (2025), restated in the notation of Section 2.1. Worker ℓ's indirect utility from firm i in market j is $u_{\ell ij} = w_i\, z_{\ell i}\, v_{\ell j}$, with the taste draws following Fréchet distributions,

$$\mathbb{P}(z_{\ell i} \le z) = e^{-T_i\, z^{-\varsigma}}, \qquad \mathbb{P}(v_{\ell j} \le v) = e^{-v^{-\eta}},$$

independent across workers, firms, and markets, and of each other. The worker first observes the market taste draws $\{v_{\ell j}\}$ and chooses a market; she then observes the firm taste draws within the chosen market and chooses a firm. The unconditional probability that she works at firm i of market j is therefore

$$\chi_i = \underbrace{\mathbb{P}\Big(w_i z_{\ell i} \ge \max_{i' \ne i} w_{i'} z_{\ell i'}\Big)}_{\text{A. firm choice within } j} \times \underbrace{\mathbb{P}\Big[\mathbb{E}_j\big(\max_i w_i z_{\ell i}\big)\, v_{\ell j} \ge \max_{j' \ne j} \mathbb{E}_{j'}\big(\max_i w_i z_{\ell i}\big)\, v_{\ell j'}\Big]}_{\text{B. market choice}}.$$

Step A: the firm choice. The utility offer of firm i has distribution

$$G_i(\vartheta) = \mathbb{P}\big[w_i z_{\ell i} \le \vartheta\big] = \mathbb{P}\Big[z_{\ell i} \le \frac{\vartheta}{w_i}\Big] = e^{-T_i w_i^{\varsigma} \vartheta^{-\varsigma}}, \qquad g_i(\vartheta) = T_i\, \varsigma\, w_i^{\varsigma}\, \vartheta^{-(\varsigma+1)}\, e^{-T_i w_i^{\varsigma} \vartheta^{-\varsigma}}.$$

By independence, the best rival offer has distribution

$$\mathbb{P}\Big(\max_{i' \ne i} w_{i'} z_{\ell i'} \le \vartheta\Big) = \prod_{i' \ne i} e^{-T_{i'} w_{i'}^{\varsigma} \vartheta^{-\varsigma}} = e^{-\Phi_j^{-i}\, \vartheta^{-\varsigma}}, \qquad \Phi_j^{-i} \equiv \sum_{i' \ne i} T_{i'} w_{i'}^{\varsigma},$$

and the best offer in the market, $G_j(\vartheta) = e^{-\Phi_j \vartheta^{-\varsigma}}$ with $\Phi_j = \sum_{i' \in j} T_{i'} w_{i'}^{\varsigma}$ and density

$g_j(\vartheta) = \varsigma\,\Phi_j\,\vartheta^{-(\varsigma+1)}e^{-\Phi_j\vartheta^{-\varsigma}}$. Integrating over the own offer,

$$\mathbb{P}\Big(\max_{i'\neq i} w_{i'}z_{\ell i'} \le w_i z_{\ell i}\Big) = \int_0^\infty e^{-\Phi_j^{-i}\vartheta^{-\varsigma}}\, g_i(\vartheta)\,d\vartheta = \int_0^\infty T_i\,\varsigma\,w_i^{\varsigma}\,\vartheta^{-(\varsigma+1)}\,e^{-\Phi_j\vartheta^{-\varsigma}}\,d\vartheta$$
$$= \frac{T_i w_i^{\varsigma}}{\Phi_j}\int_0^\infty \varsigma\,\Phi_j\,\vartheta^{-(\varsigma+1)}\,e^{-\Phi_j\vartheta^{-\varsigma}}\,d\vartheta = \frac{T_i w_i^{\varsigma}}{\Phi_j} = s_{i|j},$$

the last integral being that of the density g_j, hence one.

Step B: the market choice. The expected best offer in market j is

$$\mathbb{E}_j\big(\max_i w_i z_{\ell i}\big) = \int_0^\infty \vartheta\, g_j(\vartheta)\,d\vartheta = \int_0^\infty \varsigma\,\Phi_j\,\vartheta^{-\varsigma}\,e^{-\Phi_j\vartheta^{-\varsigma}}\,d\vartheta = \Phi_j^{1/\varsigma}\,\Gamma\Big(\frac{\varsigma-1}{\varsigma}\Big) = \Phi_j^{1/\varsigma}\,\Gamma,$$

by the change of variables $x = \Phi_j\vartheta^{-\varsigma}$, where $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\,dt$ is the Gamma function and $\Gamma \equiv \Gamma(1-1/\varsigma)$, requiring $\varsigma > 1$. The worker therefore compares $\Phi_j^{1/\varsigma}\,\Gamma\,v_{\ell j}$ across markets; since $v_{\ell j}$ is Fréchet with shape η, the offer $\Phi_j^{1/\varsigma}\Gamma\,v_{\ell j}$ is Fréchet with scale $\Phi_j^{\eta/\varsigma}\Gamma^{\eta}$ and shape η, and the same integral as in Step A gives

$$\mathbb{P}\Big(\Phi_j^{1/\varsigma}\Gamma\,v_{\ell j} \ge \max_{j'\neq j}\Phi_{j'}^{1/\varsigma}\Gamma\,v_{\ell j'}\Big) = \frac{\Gamma^{\eta}\,\Phi_j^{\eta/\varsigma}}{\Phi} = s_j, \qquad \Phi \equiv \sum_{j'}\Gamma^{\eta}\,\Phi_{j'}^{\eta/\varsigma}.$$

Multiplying the two steps, $\chi_i = s_{i|j}\,s_j$, and with a mass $\mathcal{L}$ of workers making i.i.d. choices, firm-level labour supply is $l_i^s(w_i) = s_{i|j}\,s_j\,\mathcal{L}$, which is (1).

The supply elasticity. Holding rivals' wages and Φ fixed per (A1), while internalising the own-wage effect on Φ_j,

$$\frac{d\log l_i^s}{d\log w_i} = \frac{d\log s_{i|j}}{d\log w_i} + \frac{d\log s_j}{d\log w_i} = \Big(\varsigma - \varsigma\,s_{i|j}\Big) + \frac{\eta}{\varsigma}\cdot\varsigma\,s_{i|j} = \varsigma\,(1-s_{i|j}) + \eta\,s_{i|j},$$

using $d\log\Phi_j/d\log w_i = \varsigma\,s_{i|j}$, which is the elasticity (6).

The workers' welfare index. The representative-household formulation of Azkarate-Askasua and Zerecero (2025) supplies the foundation for the workers' welfare index.

Consider a household that allocates its members across firms and markets at given positive wages. Let $\chi_i = l_i/\mathcal{L}$, $s_j = \sum_{i\in j}\chi_i$, and $s_{i|j} = \chi_i/s_j$. With the amenities and wage indices of (1), define

$$H_j = -\sum_{i\in j} s_{i|j}\log\frac{s_{i|j}}{T_i}, \qquad H = -\sum_j s_j \log\frac{s_j}{\Gamma^\eta}.$$

These entropy terms value diversity in the allocation of employment, with amenities shifting the relative value of each job. The household solves

$$\max_{\{\chi_i\geq 0\}:\,\sum_i\chi_i=1}\left\{\sum_i \chi_i\log w_i + \frac{1}{\varsigma}\sum_j s_j H_j + \frac{1}{\eta}H\right\}, \tag{S1.1}$$

with the usual convention $0\log 0 = 0$. Assume $\varsigma > 1$ and $0 < \eta < \varsigma$; the former makes $\Gamma = \Gamma(1-1/\varsigma)$ finite in the underlying two-step worker problem.

Proposition S1.1. *Problem* (S1.1) *has a unique solution, and this solution reproduces the labour supply* (1)*. Its maximised value is* $U^{\mathrm{RH}} = \eta^{-1}\log\Phi = \log\mathcal{W}$.

Proof. Fix a market share $s_j > 0$. The conditional shares maximise

$$\sum_{i\in j} s_{i|j}\log w_i - \frac{1}{\varsigma}\sum_{i\in j} s_{i|j}\log\frac{s_{i|j}}{T_i}, \qquad \sum_{i\in j} s_{i|j} = 1.$$

Strict concavity gives a unique interior optimum. The first-order conditions make $s_{i|j}$ proportional to $T_i w_i^\varsigma$, so adding up and substituting back gives

$$s_{i|j} = \frac{T_i w_i^\varsigma}{\Phi_j}, \qquad V_j = \frac{1}{\varsigma}\log\Phi_j, \qquad \Phi_j = \sum_{i\in j} T_i w_i^\varsigma,$$

where V_j is the optimised within-market value. The remaining problem is

$$\max_{\{s_j\geq 0\}:\,\sum_j s_j=1}\left\{\sum_j s_j V_j - \frac{1}{\eta}\sum_j s_j\log\frac{s_j}{\Gamma^\eta}\right\}.$$

Again the objective is strictly concave. Its first-order conditions make s_j proportional to $\Gamma^\eta \exp(\eta V_j)$, yielding

$$s_j = \frac{\Gamma^\eta \Phi_j^{\eta/\varsigma}}{\Phi}, \qquad \Phi = \sum_j \Gamma^\eta \Phi_j^{\eta/\varsigma}.$$

Thus $I_i = \mathcal{L} s_j s_{i|j}$ reproduces (1). At these shares, $\log(s_j/\Gamma^\eta) = \eta V_j - \log \Phi$, so substitution gives

$$U^{\mathrm{RH}} = \frac{1}{\eta} \log \Phi = \log \mathcal{W}. \tag{S1.2}$$

□

Note that this construction does not require $\eta > 1$. In the original two-step choice problem, the value of the chosen market before the firm-specific draw is realised is $\max_j \{\Gamma \Phi_j^{1/\varsigma} v_{\ell j}\}$. Its distribution function is $\exp(-\Phi u^{-\eta})$ for $u > 0$, so its quantile at probability $q \in (0,1)$ is $\mathcal{W}(-\log q)^{-1/\eta}$. All fixed quantiles, including the median, are proportional to $\mathcal{W}$. Its mean is finite only when $\eta > 1$; the representative-household value and the quantiles remain finite for every $\eta > 0$. Accordingly, $\mathcal{W}$ is a welfare index, not a claim that mean utility exists throughout the parameter space.

The workers' consumption equivalent. Scaling all wages by a common factor scales $\mathcal{W}$ and the wage bill $\mathcal{B}$ by that factor and leaves employment shares unchanged. Hence $\mathcal{N} = \mathcal{W}/\mathcal{B}$ is invariant to the wage level.

The index is also highest when all wages are equal. By (S1.2) and $\mathcal{B} = \mathcal{L} \sum_i \chi_i w_i$,

$$\log \mathcal{N} = \sum_i \chi_i \log w_i + \frac{1}{\varsigma} \sum_j s_j H_j + \frac{1}{\eta} H - \log\Big(\mathcal{L} \sum_i \chi_i w_i\Big).$$

Jensen's inequality gives $\log \sum_i \chi_i w_i \geq \sum_i \chi_i \log w_i$, with equality only when all wages are equal, so $\log \mathcal{N} \leq \varsigma^{-1} \sum_j s_j H_j + \eta^{-1} H - \log \mathcal{L}$. The right side is the

household objective at equal wages, which is maximised by $s_{i|j} \propto T_i$ and $s_j \propto (\sum_{i \in j} T_i)^{\eta/\varsigma}$, the shares that equal wages induce. The employment index therefore reaches its maximum at the allocation workers choose when wages are equal, which depends on amenities and taste draws alone, and is lower at any allocation induced by unequal wages.

Let $\mathcal{B}^{\mathrm{CE}}$ be the wage bill that, under a proportional scaling of the reference wage vector, gives the household its current utility. Since the reference employment index is $\bar{\mathcal{N}}$, (S1.2) requires

$$\log(\mathcal{B}^{\mathrm{CE}} \bar{\mathcal{N}}) = \log(\mathcal{B}\mathcal{N}), \qquad \mathcal{B}^{\mathrm{CE}} = \frac{\mathcal{B}\mathcal{N}}{\bar{\mathcal{N}}} = \frac{\sigma C \mathcal{N}}{\bar{\mathcal{N}}}.$$

This is the workers' term in (22). Multiplying household utility by the fixed worker mass $\mathcal{L}$ leaves this conversion unchanged.

The owners' consumption equivalent and social welfare. Owners have linear utility, so their consumption-equivalent welfare is aggregate profit, $\mathcal{W}^f = \Pi = (1-\sigma)C$. Social welfare (22) adds the two consumption equivalents with equal weights per unit of reference consumption.

S1.2 Section 4.3: Physical capital

Each firm spends a constant share of revenue on physical capital, and the task block receives the share κ_f/μ of gross revenue, as in the baseline:

$$p_p\, k_i^p = \frac{\kappa_p}{\mu} r_i, \qquad c_{k,i} k_i = \frac{\kappa_f}{\mu} \tilde{x}_i r_i, \qquad \omega_i l_i = \frac{\kappa_f}{\mu} (1-\tilde{x}_i) r_i. \tag{S1.3}$$

Automation changes the split of the task block between labour and automation capital, not the revenue share of physical capital. In terms of revenue net of the

physical-capital bill, $r_i^{\mathrm{N}} \equiv r_i - p_p k_i^p = (\tilde{\mu}/\mu)\, r_i$ with

$$\tilde{\mu} \;\equiv\; \mu - \kappa_p, \tag{S1.4}$$

the firm-level conditions of the baseline carry over after replacing (μ, r_i) by $(\tilde{\mu}, r_i^{\mathrm{N}})$; $\tilde{\mu}$ is a curvature, not a markup, and the restriction $\kappa < \mu$ is equivalent to $\kappa_f < \tilde{\mu}$. The markdown schedule and the automation rule (31) are unchanged.

Summing the physical-capital shares over firms and using goods-market clearing, $p_p K^p = (\kappa_p/\mu)\, Y$, and the supply curve gives the price of physical capital as a function of aggregate output,

$$p_p \;=\; \left(\frac{\kappa_p}{\mu}\, Y\right)^{1/(1+e)},$$

which is (34) in the main text. Every solve imposes (34): the estimated economy, every candidate parameter vector in estimation, and every counterfactual allocation.

With $\mathcal{E} \equiv \sum_i c_{k,i} k_i$ the aggregate automation expenditure and $\Pi_p^U \equiv p_p K^p - G_p(K^p)$ the capital producer's profit, national income and consumption are

$$Y = \mathcal{B} + \mathcal{E} + p_p K^p + \Pi, \qquad C = \mathcal{B} + \Pi + \Pi_p^U = Y - \mathcal{E} - G_p. \tag{S1.5}$$

Owner i consumes her operating profit and her dividend,

$$c_i^f \;\equiv\; \tilde{\pi}_i + \bar{s}_i^{\pi}\, \Pi_p^U, \tag{S1.6}$$

where the dividend share $\bar{s}_i^{\pi}$ is her profit share at the welfare reference allocation. The welfare measure (22) keeps its form, with $\sigma = \mathcal{B}/C$, so owners' consumption-equivalent welfare is their income, $(1-\sigma)C = \Pi + \Pi_p^U$. Dividends enter no equilibrium condition.

S2 Characterisation with two market types

This section shows that Propositions 2–4 extend to an economy with two types of local labour market, in which workers move both within and between markets.

S2.1 Environment

The economy has $J/2$ local labour markets of each type $m \in \{0, 1\}$. Each market contains two firms, H and L, with a common amenity T. Productivities are (ϕ_{Hh}, ϕ_{Lh}) in markets of type 0 and $\Psi(\phi_{Hh}, \phi_{Lh})$ in markets of type 1, with $\phi_{Hh} > \phi_{Lh} > 0$ and $\Psi > 1$. Technology, demand and the automation-capital producer are as in Section 2, and Assumption 1 and (A.3) hold. I consider equilibria that are symmetric across markets of the same type, and I assume that these equilibria exist and are differentiable along the paths considered below. As in Definition 1(ii), firms take Y and Φ as given, including along these paths.

Write $s_{i|m}$ for the within-market employment share of firm $i \in \{H, L\}$ in a market of type m, w_{im}, ω_{im}, $\tilde{x}_{im}$ and r_{im} for its wage, marginal labour cost, automated share and revenue, and Φ_m for the wage index of a type-m market. Let $\nu_m \equiv \varsigma^{-1} \log \Phi_m$, and let $\bar{s}_m \equiv (J/2)\, s_j$ for any market j of type m denote the share of total employment in type-m markets. By (1),

$$s_{i|m} = \frac{w_{im}^{\varsigma}}{w_{Hm}^{\varsigma} + w_{Lm}^{\varsigma}}, \qquad \bar{s}_m = \frac{e^{\eta \nu_m}}{e^{\eta \nu_0} + e^{\eta \nu_1}}, \qquad l_{im} = \frac{2\mathcal{L}}{J}\, \bar{s}_m\, s_{i|m}. \tag{S2.1}$$

Each firm's markdown is $\delta_{im} = \delta(s_{i|m})$ from (13). As in Section 3.2, the wage rule is indexed by $\theta \in [0, 1]$,

$$w_{im} = \left\{1 - \theta\left[1 - \delta(s_{i|m})\right]\right\} \omega_{im},$$

so that $\theta = 0$ is the equilibrium without labour markdowns and $\theta = 1$ is the wage-posting equilibrium.

Notation. As in Online Appendix A.1, $\varrho \equiv (\mu-1)/(\mu-\kappa_f)$, $h(\tilde{x})$ is the displacement elasticity (16), $g(\tilde{x})$ is the labour-demand response (27), and

$$\mathcal{F}(\tilde{x}) \equiv 1 + h(\tilde{x}) + \frac{\kappa_f}{\mu-\kappa_f}(1-\tilde{x}),$$

with $\mathcal{F}_{im} \equiv \mathcal{F}(\tilde{x}_{im})$, $g_{im} \equiv g(\tilde{x}_{im})$ and $h_{im} \equiv h(\tilde{x}_{im})$. Along any path, $z \equiv -d\log c_k$ is the fall in the automation price, $a_{im} \equiv d\log\omega_{im} + z$ is the change in firm i's marginal labour cost relative to that price, $\bar{a}_m \equiv dv_m + z$ is the corresponding change in the market wage index, and $\varpi_{im} \equiv (J/2)\, r_{im}/Y$ is the revenue weight of firm i across all type-m markets, so that $\sum_{m,i}\varpi_{im} = 1$.

S2.2 Two lemmas

Lemma S2.1 (Firm-level responses)**.** *Fix Y, the automation price c_k, a firm's marginal labour cost ω and its productivity ϕ_h.*

(i) *The automated share is $\tilde{x} = M^{-1}R^{-1}(\omega/c_k)$, and $d\tilde{x} = (1-\tilde{x})\,h(\tilde{x})\,(d\log\omega - d\log c_k)$.*

(ii) *Revenue satisfies $d\log r = \varrho\, d\log Y - \frac{\kappa_f}{\mu-\kappa_f}\big[(1-\tilde{x})\, d\log\omega + \tilde{x}\, d\log c_k\big]$.*

(iii) *Labour demand, $l^d = (\kappa_f/\mu)(1-\tilde{x})\, r/\omega$, is strictly increasing in ϕ_h and satisfies*

$$\begin{aligned} d\log l^d &= \varrho\, d\log Y - \mathcal{F}(\tilde{x})\, d\log\omega - g(\tilde{x})\, d\log c_k, \\ \mathcal{F}(\tilde{x}) + g(\tilde{x}) &= \frac{\mu}{\mu-\kappa_f} = \rho\varrho. \end{aligned} \tag{S2.2}$$

Proof. Cost minimisation assigns tasks below $R^{-1}(\omega/c_k)$ to capital and the remaining tasks to labour. The Cobb–Douglas task aggregate then gives the cost function $(y/\phi_h)^{1/\kappa_f} q(c_k,\omega)$, where $\log q$ is the average across tasks of the log cost per efficiency unit of the cheaper factor, and $d\log q = (1-\tilde{x})\, d\log\omega + \tilde{x}\, d\log c_k$ because the two factors cost the same at the threshold. Maximising $Y^{1/\rho}y^{1/\mu}$ less

this cost gives

$$r = Y^{\varrho}\,\phi_h^{1/(\mu-\kappa_f)}\left[\frac{\kappa_f}{\mu\, q(c_k,\omega)}\right]^{\kappa_f/(\mu-\kappa_f)},$$

which is part (ii) and is strictly increasing in ϕ_h. Differentiating $\log R(M\tilde{x}) = \log\omega - \log c_k$ gives part (i). Labour demand follows from (10), and differentiating $\log l^d = \log(1-\tilde{x}) + \log r - \log\omega + \text{const}$ with parts (i) and (ii) gives (S2.2). The identity holds because $1 + \kappa_f/(\mu-\kappa_f) = \mu/(\mu-\kappa_f)$ and $\rho(\mu-1) = \mu$. Lemmas 1 and 2 in the Online Appendix give the details. □

Lemma S2.2 (Aggregate restriction)**.** *Along any equilibrium path with fixed productivities,*

$$d\log l_{im} = \varrho\; d\log Y + \rho\varrho\; z - \mathcal{F}_{im}\, a_{im}, \tag{S2.3}$$

and

$$\sum_{m,i} A_{im}\, a_{im} = \varrho\left(\rho + \frac{\kappa_f}{1-\kappa_f}\right)z, \qquad A_{im} \equiv \bar{s}_m\, s_{i|m}\,\mathcal{F}_{im} + \frac{\kappa_f\varrho}{1-\kappa_f}\,\varpi_{im}\,(1-\tilde{x}_{im}) > 0. \tag{S2.4}$$

Proof. Equation (S2.3) is (S2.2) with $d\log\omega_{im} = a_{im} - z$ and $d\log c_k = -z$. Goods-market clearing gives $d\log Y = \sum_{m,i}\varpi_{im}\, d\log r_{im}$. Substituting Lemma S2.1(ii) and using $1-\varrho = (1-\kappa_f)/(\mu-\kappa_f)$,

$$d\log Y = \frac{\kappa_f}{1-\kappa_f}\left[z - \sum_{m,i}\varpi_{im}(1-\tilde{x}_{im})\, a_{im}\right]. \tag{S2.5}$$

Aggregate employment is fixed, and by (S2.1) firm i in type-m markets employs the share $\bar{s}_m s_{i|m}$ of all workers, so $\sum_{m,i}\bar{s}_m s_{i|m}\, d\log l_{im} = 0$. Weighting (S2.3) by these shares, summing, and substituting (S2.5) gives (S2.4). Each A_{im} is a sum of strictly positive terms. □

S2.3 Equilibrium ranking

Proposition S2.1. *In the wage-posting equilibrium, the more productive firm in each market type is larger, pays a higher wage, sets a lower markdown, faces a higher marginal labour cost and automates more:*

$$s_{H|m} > s_{L|m}, \quad l_{Hm} > l_{Lm}, \quad r_{Hm} > r_{Lm},$$
$$w_{Hm} > w_{Lm}, \quad \delta_{Hm} < \delta_{Lm}, \quad \omega_{Hm} > \omega_{Lm}, \quad \tilde{x}_{Hm} > \tilde{x}_{Lm}.$$

Its profit margin is higher if and only if $(1-\delta_{Hm})(1-\tilde{x}_{Hm}) > (1-\delta_{Lm})(1-\tilde{x}_{Lm})$. *The rankings of shares, employment and marginal labour costs also hold in the equilibrium without labour markdowns.*

Proof. Suppose $s_{H|m} \leq s_{L|m}$. By (S2.1), $w_{Hm} \leq w_{Lm}$, and since $\delta(\cdot)$ is strictly decreasing, $\delta_{Hm} \geq \delta_{Lm}$. Hence $\omega_{Hm} = w_{Hm}/\delta_{Hm} \leq \omega_{Lm}$. Both firms face the same Y and c_k, so by Lemma S2.1(iii), $l_{Hm} > l_{Lm}$. But $l_{Hm}/l_{Lm} = s_{H|m}/s_{L|m} \leq 1$ by (S2.1), a contradiction. Hence $s_{H|m} > s_{L|m}$ and $l_{Hm} > l_{Lm}$, and the wage, markdown and marginal-labour-cost rankings follow. Lemma S2.1(i) gives $\tilde{x}_{Hm} > \tilde{x}_{Lm}$. By (10), $r_{im} = \omega_{im} l_{im}/[(\kappa_f/\mu)(1-\tilde{x}_{im})]$, whose numerator is larger and denominator smaller for H, so $r_{Hm} > r_{Lm}$. By (17),

$$\frac{\tilde{\pi}_{im}}{r_{im}} = 1 - \frac{\kappa_f}{\mu} + \frac{\kappa_f}{\mu}(1-\delta_{im})(1-\tilde{x}_{im}),$$

which gives the profit-margin condition. Without labour markdowns, $\omega_{im} = w_{im}$, and the same argument gives the rankings of shares, employment and marginal labour costs. □

S2.4 From labour market power to automation

All coefficients in this subsection are evaluated at the equilibrium without labour markdowns, $\theta = 0$. Let $\mathfrak{d}_{im} \equiv 1 - \delta(s_{i|m})$ denote the markdown wedge that wage

posting would impose at these shares, and define

$$\mathcal{V}_m \equiv 1 - (\varsigma - \eta) \sum_i \frac{s_{i|m}}{\varsigma + \mathcal{F}_{im}}, \qquad \mathcal{X}_{im} \equiv \mathcal{V}_m \, \mathfrak{d}_{im} - \frac{\varsigma - \eta}{\varsigma} \sum_{i'} \frac{s_{i'|m} \, \mathcal{F}_{i'm} \, \mathfrak{d}_{i'm}}{\varsigma + \mathcal{F}_{i'm}},$$

and the weighted average

$$\bar{\mathcal{X}} \equiv \frac{\sum_{m,i} A_{im} \, \mathcal{X}_{im} / [\mathcal{V}_m (\varsigma + \mathcal{F}_{im})]}{\sum_{m,i} A_{im} / [\mathcal{V}_m (\varsigma + \mathcal{F}_{im})]}.$$

Since $\mathcal{F}_{im} > 0$, $\sum_i s_{i|m}/(\varsigma + \mathcal{F}_{im}) < 1/\varsigma$, so $\mathcal{V}_m > \eta/\varsigma > 0$ and the weights in $\bar{\mathcal{X}}$ are positive. $\mathcal{X}_{im}$ measures firm i's exposure to the introduction of wage-setting power: its own wedge, scaled by $\mathcal{V}_m$, less a term that captures how its market's wage index responds to both firms' wedges.

Proposition S2.2. *Suppose*

$$\max_m \mathcal{X}_{Lm} < \bar{\mathcal{X}} < \min_m \mathcal{X}_{Hm}. \tag{S2.6}$$

Then, in both market types, a marginal introduction of wage-setting power, $d\theta > 0$ at $\theta = 0$, lowers the productive firm's within-market employment share and its wage premium w_{Hm}/w_{Lm}, raises the marginal labour cost and automation at H, lowers both at L, and widens the gap in the shares of tasks that labour performs:

$$\frac{d}{d\theta}\Big[\log(1 - \tilde{x}_{Lm}) - \log(1 - \tilde{x}_{Hm})\Big] = h_{Hm} \frac{d \log \omega_{Hm}}{d\theta} + h_{Lm} \left| \frac{d \log \omega_{Lm}}{d\theta} \right| > 0.$$

Condition (S2.6) requires that, in every market type, the productive firm's exposure exceed the economy-wide average and the less productive firm's fall below it. At the symmetric equilibrium of the identical-market benchmark, $\Psi = 1$, condition (S2.6) holds. Corresponding firms have identical allocations across market types, so $\bar{\mathcal{X}}$ is a weighted average of $\mathcal{X}_{H0}$ and $\mathcal{X}_{L0}$ with positive weights, and $\mathcal{X}_{H0} - \mathcal{X}_{L0} = \mathcal{V}_0(\mathfrak{d}_{H0} - \mathfrak{d}_{L0}) > 0$ because $s_{H|0} > s_{L|0}$ by Proposition S2.1.

Proof. All differentials are taken at $\theta = 0$ with c_k fixed, so $z = 0$, $a_{im} = d\log\omega_{im}$ and $\bar{a}_m = dv_m$. By the wage rule, $d\log w_{im} = a_{im} - \mathfrak{d}_{im}\,d\theta$, because the term in $ds_{i|m}$ is multiplied by θ. Since $\Phi_m = T\sum_i w_{im}^{\varsigma}$, $\bar{a}_m = \sum_i s_{i|m}\,d\log w_{im}$. Differentiating (S2.1),

$$d\log l_{im} = \varsigma\left(a_{im} - \mathfrak{d}_{im}\,d\theta\right) - (\varsigma - \eta)\,\bar{a}_m - \eta\sum_n \bar{s}_n\,\bar{a}_n.$$

Equating this with (S2.3) gives

$$(\varsigma + \mathcal{F}_{im})\,a_{im} = Q + (\varsigma - \eta)\,\bar{a}_m + \varsigma\,\mathfrak{d}_{im}\,d\theta, \qquad Q \equiv \varrho\,d\log Y + \rho\varrho\,z + \eta\sum_n \bar{s}_n\,\bar{a}_n, \tag{S2.7}$$

where Q is common to all firms. The term $\rho\varrho\,z$ is zero here; it is kept so that (S2.7) also serves the proof of Proposition S2.3, where $z \neq 0$. Substituting (S2.7) into $\bar{a}_m = \sum_i s_{i|m}(a_{im} - \mathfrak{d}_{im}\,d\theta)$, solving for $\bar{a}_m$, and substituting back,

$$a_{im} = \frac{Q + \varsigma\,\mathcal{X}_{im}\,d\theta}{\mathcal{V}_m(\varsigma + \mathcal{F}_{im})}.$$

With $z = 0$, Lemma S2.2 gives $\sum_{m,i} A_{im}a_{im} = 0$, so $Q = -\varsigma\bar{\mathcal{X}}\,d\theta$ and

$$a_{im} = \frac{\varsigma\,(\mathcal{X}_{im} - \bar{\mathcal{X}})}{\mathcal{V}_m(\varsigma + \mathcal{F}_{im})}\,d\theta. \tag{S2.8}$$

Condition (S2.6) gives $a_{Hm} > 0 > a_{Lm}$ in both market types. Subtracting (S2.3) for L from that for H in the same market, and using $l_{Hm}/l_{Lm} = s_{H|m}/s_{L|m}$ and $ds_{L|m} = -ds_{H|m}$,

$$\frac{ds_{H|m}}{s_{H|m}\,s_{L|m}} = \mathcal{F}_{Lm}\,a_{Lm} - \mathcal{F}_{Hm}\,a_{Hm} < 0.$$

By (S2.1), $d\log(w_{Hm}/w_{Lm}) = ds_{H|m}/(\varsigma\,s_{H|m}s_{L|m}) < 0$, so the wage premium falls. Lemma S2.1(i) gives $d\tilde{x}_{im} = (1 - \tilde{x}_{im})\,h_{im}\,a_{im}$ and $d\log(1 - \tilde{x}_{im}) = -h_{im}\,a_{im}$, so automation rises at H and falls at L, and subtracting the two firms gives the displayed expression. □

Unlike in the analytical economy, employment levels need not move with

within-market shares, because workers also move between market types. By (S2.3) and (S2.5) with $z = 0$,

$$\frac{d \log l_{im}}{d\theta} = \varrho \, \frac{d \log Y}{d\theta} - \mathcal{F}_{im} \, \frac{d \log \omega_{im}}{d\theta},$$
$$\varrho \, \frac{d \log Y}{d\theta} = -\frac{\kappa_f \varrho}{1 - \kappa_f} \sum_{m,i} \varpi_{im} (1 - \tilde{x}_{im}) \, \frac{d \log \omega_{im}}{d\theta}.$$

Employment therefore falls at every productive firm and rises at every less productive firm if and only if

$$\max_m \mathcal{F}_{Lm} \, \frac{d \log \omega_{Lm}}{d\theta} < \varrho \, \frac{d \log Y}{d\theta} < \min_m \mathcal{F}_{Hm} \, \frac{d \log \omega_{Hm}}{d\theta}.$$

S2.5 From automation to labour market power

For each market type, define $\Delta_{\Upsilon,m} \equiv g_{Hm} - g_{Lm}$, the analogue of (28).

Proposition S2.3. *In the wage-posting equilibrium, a marginal fall in the price of automation capital raises automation at every firm. Within each market type,* $\operatorname{sign}(ds_{H|m}) = \operatorname{sign}(\Delta_{\Upsilon,m})$. *If* $\Delta_{\Upsilon,m} > 0$, *the productive firm's markdown falls, the less productive firm's markdown rises, and the markdown gap* $\delta_{Lm} - \delta_{Hm}$ *widens. These movements reverse if* $\Delta_{\Upsilon,m} < 0$, *and shares and markdowns do not change to first order if* $\Delta_{\Upsilon,m} = 0$.

Proof. Let $z > 0$ and write $ds_m \equiv ds_{H|m} = -ds_{L|m}$. By (S2.1), $d \log w_{im} = dv_m + \varsigma^{-1} d \log s_{i|m}$, and the wage-posting rule $\omega_{im} = w_{im}/\delta(s_{i|m})$ adds the markdown response. Hence

$$a_{Hm} = \bar{a}_m + \mathcal{D}_{Hm} \, ds_m, \qquad a_{Lm} = \bar{a}_m - \mathcal{D}_{Lm} \, ds_m, \qquad \mathcal{D}_{im} \equiv \frac{1}{\varsigma \, s_{i|m}} - \frac{\delta'(s_{i|m})}{\delta_{im}} > 0,$$

where $\mathcal{D}_{im} > 0$ by (A.1). Subtracting (S2.3) for the two firms in the same market,

$$\frac{ds_m}{s_{H|m} \, s_{L|m}} = \mathcal{F}_{Lm} \, a_{Lm} - \mathcal{F}_{Hm} \, a_{Hm}. \tag{S2.9}$$

With $\mathcal{D}_m \equiv \mathcal{D}_{Hm} + \mathcal{D}_{Lm}$, the first two equations give $a_{Hm} - a_{Lm} = \mathcal{D}_m\, ds_m$, and substituting into (S2.9),

$$\left(1 + \mathcal{D}_m s_{H|m} s_{L|m} \mathcal{F}_{Hm}\right) a_{Hm} = \left(1 + \mathcal{D}_m s_{H|m} s_{L|m} \mathcal{F}_{Lm}\right) a_{Lm},$$
$$\bar{a}_m = \frac{\mathcal{D}_{Lm}\, a_{Hm} + \mathcal{D}_{Hm}\, a_{Lm}}{\mathcal{D}_m}.$$

Hence $a_{im} = \varkappa_{im}\, \bar{a}_m$ for coefficients $\varkappa_{im} > 0$ that depend only on the initial equilibrium.

Within each market, $\sum_i s_{i|m}\, d \log s_{i|m} = 0$, so weighting (S2.3) by $s_{i|m}$ and summing gives the change in employment in type-m markets. By (S2.1), this change is $d \log \bar{s}_m = \eta\, \bar{a}_m - \eta \sum_n \bar{s}_n \bar{a}_n$. Equating the two, with $\bar{\mathcal{F}}_m \equiv \sum_i s_{i|m} \mathcal{F}_{im} \varkappa_{im} > 0$ and Q from (S2.7),

$$(\eta + \bar{\mathcal{F}}_m)\, \bar{a}_m = Q.$$

Substituting $a_{im} = \varkappa_{im} Q / (\eta + \bar{\mathcal{F}}_m)$ into Lemma S2.2,

$$Q \sum_{m,i} \frac{A_{im}\, \varkappa_{im}}{\eta + \bar{\mathcal{F}}_m} = \varrho \left(\rho + \frac{\kappa_f}{1 - \kappa_f}\right) z > 0.$$

Hence $Q > 0$, so $\bar{a}_m > 0$ and $a_{im} > 0$ at every firm, and Lemma S2.1(i) gives $d\tilde{x}_{im} = (1 - \tilde{x}_{im})\, h_{im}\, a_{im} > 0$.

Finally, substituting $a_{Hm} = a_{Lm} + \mathcal{D}_m\, ds_m$ into (S2.9) and using $\mathcal{F}_{Lm} - \mathcal{F}_{Hm} = g_{Hm} - g_{Lm}$ from (S2.2),

$$\left(\frac{1}{s_{H|m}\, s_{L|m}} + \mathcal{D}_m\, \mathcal{F}_{Hm}\right) ds_m = \Delta_{\Upsilon,m}\, a_{Lm}.$$

The coefficient on the left and a_{Lm} are positive, so $\operatorname{sign}(ds_m) = \operatorname{sign}(\Delta_{\Upsilon,m})$. Since $d\delta_{Hm} = \delta'(s_{H|m})\, ds_m$ and $d\delta_{Lm} = -\delta'(s_{L|m})\, ds_m$ with $\delta' < 0$, $ds_m > 0$ lowers δ_{Hm}, raises δ_{Lm} and widens the markdown gap, and these movements reverse when $ds_m < 0$. □

S3 Normalisations

The moments used in estimation describe differences across firms and patterns of automation adoption, but do not pin down the model's scale. The normalisations in Table 1 fix this scale without changing employment shares, markdowns, automation decisions or expenditure shares. Proposition S3.1 establishes these results.

Proposition S3.1 (Normalisations). *Within the parametric family of Section 5.3, the following normalisations can be imposed jointly:*

(i) $M = 1$, $A_p = 1$, *and* $\mathcal{L}$ *equal to sample employment, fixing task units, physical-capital units and the employment scale;*

(ii) $\mathbb{E}[\log T_i] = 0$, $\mathbb{E}[\phi_{ih}] = 1$, *and* $\mathbb{E}[b_i] = 1$, *implemented by* $x_{\min} = e^{-1/\iota}$, $\mu_\phi = -\sigma_\phi^2/2$, *and* $\mu_b = -\upsilon^2/2$;

(iii) *the common task-schedule scale* $\zeta = 1$ *and comparative-advantage intercept* $R(0) = 1$. *Allowing separate power exponents* a_l *and* a_k *in the task schedules does not enlarge the set of observable outcomes: the common exponent can be set to* $a_l = a_k \equiv a$.

These normalisations leave the dispersions, the productivity–amenity dependence and the shape parameters (a, β_R) *unrestricted.*

Proof. Task units enter only through τ/M and x_i/M, so $M = 1$ is a relabelling. A common amenity multiplier cancels from both employment shares in (1) and from welfare ratios. Likewise, multiplying every b_i by a constant and dividing c_k by it leaves each installed-capital price $c_k b_i$ unchanged. The distributional means give the stated formulas for $x_{\min}$ and μ_b; the copula leaves these marginals unchanged.

For the task schedules, write $u = \tau/M$ and allow

$$\phi_l(\tau) = \zeta_l u^{a_l} e^{\beta_R u}, \qquad \phi_k(\tau) = \zeta_k u^{a_l} (1-u)^{a_k}.$$

The common factor u^{a_l} cancels from comparative advantage and contributes only

$a_l \int_0^1 \log u \, du = -a_l$ to $\log \tilde{\phi}_i$. Setting $a_l = a_k$ is therefore offset by a shift in μ_ϕ. Multiplying ζ_k and c_k by the same constant, and dividing automation quantities by it, preserves effective inputs and expenditures; this permits $\zeta_k = \zeta_l$, hence $R(0) = 1$. The remaining common scale enters productivity only through $\mu_\phi/\kappa_f + \log \zeta$ and can be set to one.

To verify the remaining level conventions, restore the physical-capital supply scale $p_p = A_p (K^p)^{1/e}$. For any $h > 0$, changing units as

$$(k_i^p, p_p, A_p, \phi_{ih}) \longmapsto (k_i^p/h,\ h p_p,\ h^{1+1/e} A_p,\ h^{\kappa_p} \phi_{ih})$$

preserves output, expenditures and resource costs. Separately, multiplying every ϕ_{ih} by $q > 0$ scales output, revenues, wages, both capital prices and A_p by q, leaving input quantities and all targeted moments unchanged. Together they scale $\mathbb{E}[\phi_{ih}]$ by $q h^{\kappa_p}$ and A_p by $q h^{1+1/e}$; because $\kappa_p < 1 < 1 + 1/e$, a unique pair (h, q) sets $A_p = 1$ and $\mathbb{E}[\phi_{ih}] = 1$. Finally, multiplying $\mathcal{L}$ and all input quantities by $\lambda > 0$, all wages and input prices by $\lambda^{\kappa-1}$, and A_p by $\lambda^{\kappa-1-1/e}$ preserves shares and automation choices. The preceding transformations restore the two level conventions. Thus $\mathcal{L}$ can match sample employment without changing the targeted moments. □

S4 Data

S4.1 Data, sample construction, and sample overview

S4.1.1 Datasets

I construct a firm-year panel of French manufacturing firms for 2009–2019 from four administrative datasets. They share a firm identifier, the SIREN number, so each firm's production, workforce, and imports can be followed in one panel. The panel starts in 2009, when the product-level production data become available.

Balance sheets. The *Fichier approché des résultats d'Esane* (FARE) provides balance sheets for French firms. I use its revenue, value added, physical capital, intermediate inputs, employment, labour costs, location, and industry.

Production survey. The *Enquête Annuelle de Production* (EAP) reports revenue and quantities by product for manufacturing firms. It covers every firm with at least 20 employees or turnover above €5 million, and a representative sample of smaller firms. Because it reports prices and quantities separately, it gives firm-level output prices (Section S4.2.3).

Workforce records. The *Déclaration Annuelle des Données Sociales* (DADS Postes) reports, for every job, the worker's wage, hours, age, occupation, and commuting zone. I aggregate the jobs to the firm-year. The records describe each firm's workforce in each year; they do not follow workers across years.

Customs records. Customs records report each firm's imports by product and year from 1996 to 2019, allowing me to trace its history of automation imports from 1996 onward. They measure automation capital (Section S4.1.2).

S4.1.2 Automation capital

Classification. Following Acemoglu and Restrepo (2022), an import is automation capital when its product belongs to a list of 141 six-digit products of the 1992 Harmonised System (HS92). The list has eight categories: industrial robots (1 code), numerically controlled machines (12), automatic machine tools (45), automatic welding equipment (2), weaving machinery (13), dedicated textile machinery (25), conveyors and transfer equipment (4), and regulating and control instruments (39). It excludes hand-operated machine tools, machine tools classified as not numerically controlled, household appliances, and information-technology equipment. The codes are listed at the end of this subsection.

From Customs products to the classification. Customs record products in the eight-digit Combined Nomenclature (CN8), which changes from year to year. For each year from 1996 to 2019, I map that year's CN8 codes to HS92 through the correspondence tables between the CN8 nomenclature in force and the HS revision of that year. I keep only codes assigned to a single HS92 product, so no import is counted twice. I keep imports of non-French origin only.[1] The measure records the automation equipment a firm imports itself; equipment bought from domestic producers or through domestic intermediaries is not observed.

The automation stock. Let $e_{it} \geq 0$ denote firm i's imports of automation capital in year t, deflated to 2015 euros with the French national-accounts investment deflator for machinery and equipment. A sample firm with no automation import recorded in a year has $e_{it} = 0$. I accumulate the flows into the stock (35) of Section 5.1, depreciating each purchase linearly to zero over eight years:

$$K_{it}^{A} = \sum_{s=0}^{7} \left(1 - \frac{s}{8}\right) e_{i,t-s}.$$

Each purchase enters the stock at its full value, loses one eighth of that value each year, and leaves after eight years.[2]

Adoption and intensity. A firm is an adopter in year t when it holds a positive stock:

$$d_{it} = 1\{K_{it}^{A} > 0\}. \tag{S4.1}$$

Adoption is not an absorbing state: a firm stops counting as an adopter once all its purchases have aged out of the eight-year window. Among adopters, automation

[1] Intra-EU records are restricted to ordinary import regimes, and special-use CN8 codes, such as grouped or simplified declarations, are excluded.

[2] I use a stock rather than the annual flow because automation projects can involve several purchases spread across fiscal years, and because equipment installed in one year remains in use for several more. Eight years is a standard accounting depreciation period for industrial machinery.

intensity is the stock relative to value added in the previous year, $K^A_{it}/\text{VA}_{i,t-1}$.

The 141 products. The six-digit HS92 codes, by category, are as follows.
Industrial robots: 847989.
Numerically controlled machines: 845811, 845891, 845921, 845931, 845951, 845961, 846011, 846021, 846031, 846221, 846231, 846241.
Automatic machine tools: 845610, 845620, 845630, 845690, 845910, 845940, 845970, 846040, 846090, 846110, 846120, 846130, 846140, 846150, 846190, 846210, 846291, 846299, 846310, 846320, 846330, 846390, 846410, 846420, 846490, 846510, 846591, 846592, 846593, 846594, 846595, 846596, 846599, 846610, 846620, 846630, 846691, 846692, 846693, 846694, 846820, 846880, 846890, 851511, 851519.
Automatic welding equipment: 851521, 851531.
Weaving machinery: 844610, 844621, 844629, 844630, 844711, 844712, 844720, 844790, 844841, 844842, 844849, 844851, 844859.
Dedicated textile machinery: 844400, 844511, 844512, 844513, 844519, 844520, 844530, 844540, 844590, 844811, 844819, 844820, 844831, 844832, 844833, 844839, 844900, 845221, 845230, 845240, 845290, 845310, 845320, 845380, 845390.
Conveyors and transfer equipment: 842832, 842833, 842839, 845730.
Regulating and control instruments: 902511, 902519, 902580, 902590, 902610, 902620, 902680, 902690, 902710, 902720, 902730, 902750, 902780, 902790, 902810, 902820, 902830, 902890, 902910, 902920, 902990, 903010, 903020, 903031, 903039, 903040, 903081, 903089, 903090, 903110, 903120, 903140, 903180, 903190, 903210, 903220, 903281, 903289, 903290.

S4.1.3 Sample construction

I prepare each dataset separately, merge the balance sheets, the production survey, and the workforce records by firm and year, and apply the sample restrictions. I then append each firm's Customs history to compute the automation capital stock.

Balance sheets. I keep manufacturing firms, with two-digit industry codes from 10 to 33, from 2009 to 2019. I remove duplicate observations on the identifier, industry, revenue, value added, capital, investment, and labour costs, and then drop firms that still appear more than once in a year.

Production survey. I construct firm-level output prices and quantities following De Ridder et al. (2026) (Section S4.2.3).

Workforce records. I drop jobs with a missing or invalid identifier, including empty strings and "000000000", workers outside ages 16–65, negative or missing gross wages, missing commuting zones, and jobs in agricultural or public-sector occupations or with no occupation code, and aggregate hours and gross wages to the firm-year.

Customs records. I pad identifiers with leading zeros to nine digits and remove the placeholder identifiers 000000000, 111111111, 222222222, 202020202, 777777777, 888888888 and 999999999. I sum the automation imports of Section S4.1.2 to the firm-year.

Merge and restrictions. I merge the balance sheets with the production survey and keep firms present in both. I then merge the workforce records by identifier and year, and assign each firm the commuting zone where its average annual hours are largest. I apply the following restrictions in order.

- *Industries.* I drop food and beverages (10 and 11), coke and refined petroleum (19), and pharmaceuticals (21), which have very few observations, and installation and repair of machinery (33), whose firms may be resellers of automation equipment rather than its users. This leaves eighteen industries, since the panel has no tobacco firms (12).

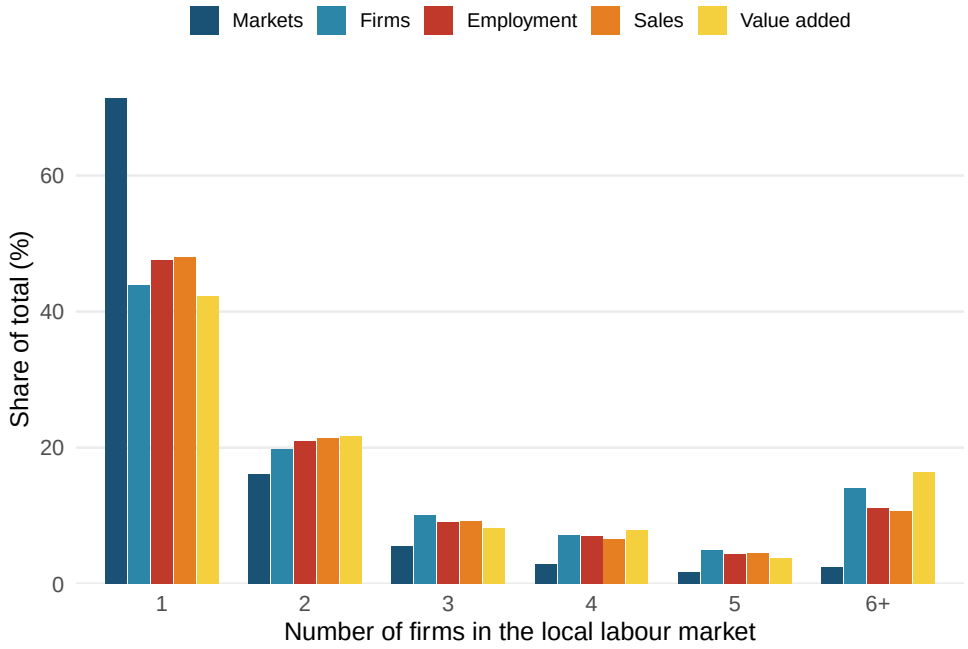
- *Missing data.* I drop observations with missing or non-positive revenue, physical output, capital, hours, employment, intermediate inputs, output prices, labour costs, wages, or commuting zone, or with a missing industry or location code, and keep one record per firm and year.
- *Consecutive years.* Firms may enter and leave the sample, but each firm must be observed in consecutive years. For a firm with gaps, I keep its longest run of consecutive years, and I drop firms whose longest run is shorter than three years.
- *Size.* I keep firms with at least five employees on average over their time in the sample.
- *Outliers.* I winsorise revenue, output prices and quantities, capital, intermediate inputs, hours, employment, gross wages, wages, and the material and wage shares at the 1st and 99th percentiles.

I fix the sample at this point and append each firm's 1996–2019 automation imports. All monetary variables are in 2015 euros, using the deflators of Section S4.2.2. The production-function estimation of Section S4.3 uses the twelve two-digit industries with the largest shares of automation imports, shown in bold in Table S4.1; I call them the production-function sample.

S4.1.4 Sample overview

The panel has 21,918 firms; the production-function sample has 134,762 firm-years for 16,525 firms. Table S4.1 lists every two-digit manufacturing industry in order of its share of manufacturing automation imports. The twelve selected industries account for 96.4 per cent of automation imports, 86.0 per cent of revenue, and 84.0 per cent of hours worked in the eighteen manufacturing industries of the panel.

Figure S4.1: DISTRIBUTION OF FIRMS ACROSS LOCAL LABOUR MARKETS



Notes: The figure groups local labour markets (five-digit industry × commuting zone) by the number of firms they contain and plots, for each group, its share of all markets, of all firms, of total employment (worker headcounts), of total sales, and of total value added. Shares within each series sum to one.

Automation is common but lumpy. In 2019, 44 per cent of the panel's firms hold a positive automation stock, while in a typical year about 30 per cent of firms in the production-function sample import automation capital.

Table S4.2 describes the distribution of firm characteristics in the production-function sample. The median firm has revenue of €4.31 million, 25 workers, and a wage bill of €0.87 million. At the ninetieth percentile the figures are €41.39 million, 168 workers, and €6.88 million.

Figure S4.1 shows how markets, firms, employment, sales and value added are distributed across local labour markets by the number of firms they contain; Section 5.4 summarises it.

Table S4.1: ALL 2-DIGIT MANUFACTURING SECTORS

Code	Sector	Firms	Revenue share	Hours share	Automation expenditure share
Panel A. Selected industries					
26	Computer and optical products	905	5.76%	6.02%	29.28%
28	Machinery	2,362	12.54%	12.78%	20.28%
29	Motor vehicles	856	7.54%	6.97%	13.65%
30	Other transport equipment	252	2.74%	2.42%	8.86%
25	Fabricated metal products	4,445	9.40%	12.02%	6.25%
27	Electrical equipment	1,033	6.91%	6.85%	5.37%
20	Chemicals	1,182	13.59%	9.11%	2.89%
22	Rubber and plastic products	2,346	9.75%	10.66%	2.82%
32	Other manufacturing	626	2.41%	2.87%	2.08%
24	Basic metals	601	6.13%	5.00%	1.89%
23	Other non-metallic mineral products	1,642	7.26%	7.00%	1.50%
13	Textiles	923	2.00%	2.27%	1.50%
	Total	16,525	86.02%	83.97%	96.35%
Panel B. Excluded industries					
16	Wood products	1,268	2.34%	2.79%	1.48%
31	Furniture	1,182	2.11%	3.07%	1.12%
17	Paper	908	5.77%	5.00%	0.54%
14	Wearing apparel	710	1.88%	2.40%	0.25%
15	Leather	309	1.05%	1.45%	0.22%
18	Printing	1,349	0.82%	1.32%	0.04%

Notes: Shares are of the eighteen-industry panel after the restrictions of Section S4.1.3, before the industry selection. Totals are computed from unrounded shares. Hours are from the workforce records. A firm that changes two-digit industry during the panel is counted in each industry it appears in, so the industry counts sum to more than the 16,525 selected firms. The industries in Panel A form the production-function sample. The automation shares use the product list of Section S4.1.2.

S4.2 Data sources and variable definitions

S4.2.1 Variables

The four datasets are linked through the SIREN identifier. The variables below enter the production-function estimation, the automation measure, and the moments of the quantification.

1. **FARE**

Table S4.2: SUMMARY STATISTICS OF OBSERVED FIRM CHARACTERISTICS

	Revenue (million €)	Fixed assets (million €)	Wage bill (DADS gross) (million €)	Employment	Log price	Log price (5-digit)
5th percentile	0.65	0.11	0.17	6.00	-3.04	-2.21
10th percentile	0.91	0.18	0.22	7.00	-1.89	-1.42
25th percentile	1.74	0.43	0.39	12.00	-0.44	-0.39
Median	4.31	1.30	0.87	25.00	0.43	0.08
Mean	19.37	9.90	3.12	73.81	0.00	0.00
75th percentile	12.64	4.78	2.27	60.00	0.74	0.53
90th percentile	41.39	19.60	6.88	168.00	1.12	1.27
95th percentile	82.83	47.89	13.37	312.00	1.65	1.80
Std. deviation	49.85	29.64	7.27	150.70	1.37	1.19

Notes: The table covers the firm-years of the production-function sample. The first "Log price" column reports log output prices after winsorisation and mean normalisation, across all firms in the sample. The second also demeans them by five-digit-industry by year fixed effects.

- **Revenue:** total revenue, `REDI_R310`.
- **Value added:** revenue plus capitalised and stocked production (`REDI_R310` + `REDI_R312` + `REDI_R311`), minus purchases of goods, raw materials and other purchases (`REDI_R210` + `REDI_R212` + `REDI_R214`) and the changes in their stocks (`REDI_R211` + `REDI_R213`); the denominator of automation intensity.
- **Capital:** total fixed physical assets, `IMMO_CORP`.
- **Intermediate inputs:** materials bought for production (`REDI_R212`) and goods bought for resale (`REDI_R210`), net of the changes in their stocks (`REDI_R213` and `REDI_R211`): (`REDI_R210` + `REDI_R212`) − (`REDI_R211` + `REDI_R213`).
- **Employment:** the firm's number of workers. It defines employment shares in local labour markets and enters the moments.
- **Labour cost:** gross wages and salaries, including base pay, overtime,

and bonuses, plus employer social security contributions: REDI_R216 + REDI_R217.

- **Firm age:** from the creation date, ENT_CRE_DAAAAMMJJ.
- **Location:** the five-digit commune code of the registered head office, DEPCOM, whose first two digits give the department. It is the parent company's location, not that of every establishment.
- **Industry:** the five-digit APE code of the firm's main activity, in revision 2 of the French NAF classification. Manufacturing is two-digit codes 10 to 33.

2. **EAP**

   - **Output prices:** built from the ten-digit product code (PRODFRA), the quantity and revenue indicators (C_UNITE_VAR), and the values in quantity and revenue (VAL_REF), as described in Section S4.2.3.

3. **DADS Postes**

   - **Hours:** annual hours of each job, NBHEUR.
   - **Wages:** gross annual earnings of each job, S_BRUT, excluding employer social security contributions.
   - **Commuting zone:** ZEMPT. A local labour market is a five-digit industry by commuting zone.

4. **Customs.** I map Customs product codes to HS92 with the correspondence tables of Bergounhon et al. (2018).

   - **Product:** the eight-digit Combined Nomenclature code, NC8, mapped to HS92 (Section S4.1.2).
   - **Origin:** the country of origin, PYOD.
   - **Import value:** VALSTAT.

S4.2.2 Deflators

I convert nominal values to 2015 euros with yearly price indices from EU KLEMS: the gross-output price index (`GO_PI`) for revenue, the intermediate-input price index (`II_PI`) for intermediate inputs, the value-added price index (`VA_PI`) for value added, and the investment price index (`Ip_GFCF`) for capital. Wages, labour costs, and output prices are deflated with the consumer price index. Automation imports are deflated with the French national-accounts investment deflator for machinery and equipment.

S4.2.3 Firm-level output prices

Following De Ridder et al. (2026) and Wong (2026), I build a firm-level output price from the EAP in three steps. Let $\mathcal{G}_{it}$ be the set of products firm i sells in year t, $\mathcal{I}_{nt} \subseteq \mathcal{I}$ the set of firms that sell product n, and P_{int} and Quantity_{int} the price and quantity of product n at firm i.

Step 1. The price of each product at each firm is its revenue divided by its quantity, $P_{int} = \text{Revenue}_{int}/\text{Quantity}_{int}$.

Step 2. Each price is divided by the average price of the product across the firms that sell it,

$$\overline{P}_{nt} = \frac{1}{|\mathcal{I}_{nt}|} \sum_{i' \in \mathcal{I}_{nt}} P_{i'nt}.$$

A product is a ten-digit product code together with its unit of measurement.

Step 3. The firm's price is the revenue-weighted average of its relative prices,

$$P_{it} = \sum_{n \in \mathcal{G}_{it}} s^R_{int} \frac{P_{int}}{\overline{P}_{nt}}, \qquad s^R_{int} = \frac{\text{Revenue}_{int}}{\sum_{n' \in \mathcal{G}_{it}} \text{Revenue}_{in't}}.$$

A firm whose products sell above their market averages has a high price, with each product weighted by its share of the firm's revenue. The log output price p_{it} is

the logarithm of P_{it} deflated by the consumer price index, and physical output is $q_{it} = r_{it} - p_{it}$.

S4.3 Markdown estimation

S4.3.1 Markdowns from production functions

The markdown is the ratio of the wage to the marginal revenue product of labour, the object $\delta_i = w_i/\omega_i$ of the model. Following Yeh et al. (2022), it can be written in terms of observed shares and two output elasticities:

$$\delta_{it} = \frac{\mu_{it}}{\theta^L_{it}} s^L_{it}, \qquad \mu_{it} = \theta^M_{it} \frac{\text{Revenue}_{it}}{\text{Intermediate-input expenditure}_{it}}, \tag{S4.2}$$

where s^L_{it} is the wage bill divided by revenue, with revenue as observed rather than corrected for the measurement error removed in Step 1 below, μ_{it} is the price-cost markup, and θ^L_{it} and θ^M_{it} are the output elasticities of labour and intermediate inputs. A markdown below one means the firm pays workers less than their marginal revenue product. The wage bill is DADS gross earnings (`S_BRUT`), which exclude employer social security contributions; the same wage bill gives the average hourly wage behind efficiency hours and the wage moments of the estimation.

Separating the markdown from the markup requires three assumptions, which I maintain throughout: firms take the prices of intermediate inputs as given; capital is predetermined and dynamic; and labour and intermediate inputs are static. Revenue, the wage bill, and intermediate-input expenditure are observed. The two output elasticities are not, and I estimate them from production functions.

S4.3.2 Production-function estimation

Specification. For each two-digit industry, output of firm i in year t is

$$y_{it} = f(k_{it}, l_{it}, m_{it}; \beta) + \varphi_{it}, \tag{S4.3}$$

where lower-case letters are logarithms of output y, capital k, labour l, and intermediate inputs m, f is translog, and φ_{it} is log Hicks-neutral productivity. The output elasticities are $\theta^L_{it} = \partial y_{it}/\partial l_{it}$ and $\theta^M_{it} = \partial y_{it}/\partial m_{it}$. Productivity follows a first-order Markov process,

$$\varphi_{it} = \mathcal{M}(\varphi_{i,t-1}) + \varepsilon^{\varphi}_{it}. \tag{S4.4}$$

A tilde denotes log expenditure, the sum of the log price and the log quantity, as in $\widetilde{k}_{it} = k_{it} + p_{k,it}$.

Labour in efficiency hours. Raw hours mix the quantity of labour with its quality, so a change in skill mix would show up as a change in productivity (Chan et al., 2026). I therefore measure labour in efficiency hours. Let $\bar{w}_{it}$ be the firm's average hourly wage, its wage bill divided by its hours. I regress its logarithm on five-digit-industry by year and commuting-zone by year fixed effects,

$$\log \bar{w}_{it} = \Lambda_{n(i)t} + \Xi_{c(i)t} + \varepsilon^{w}_{it},$$

where $n(i)$ is the firm's five-digit industry and $c(i)$ its commuting zone, and scale hours by the exponential of the residual:

$$L^{\text{eff}}_{it} = L^{\text{hours}}_{it} \exp\big(\hat{\varepsilon}^{w}_{it}\big).$$

A firm that pays more per hour than its industry and location predict is counted as employing more efficiency units per hour. Because the residual also contains within-market wage differences from productivity, amenities and markdowns, efficiency hours count part of these differences as differences in labour quality.

Three biases. Three variables the econometrician does not observe bias the estimated elasticities. Writing revenue in terms of what is observed makes them explicit:

$$\begin{aligned} r_{it} &= p_{it} + q_{it}, \qquad q_{it} \equiv y_{it} + \varepsilon^{q}_{it} \\ &= p_{it} + f(k_{it}, l_{it}, m_{it}; \beta) + \varphi_{it} + \varepsilon^{q}_{it} \\ &= p_{it} + f(\widetilde{k}_{it}, l_{it}, \widetilde{m}_{it}; \beta) + B(\boldsymbol{p}_{xit}, \widetilde{\boldsymbol{x}}_{it}, l_{it}; \beta, \boldsymbol{\vartheta}) + \varphi_{it} + \varepsilon^{q}_{it}, \end{aligned} \tag{S4.5}$$

where p_{it} is the log output price, q_{it} is measured output, ε^{q}_{it} is measurement error, $\widetilde{\boldsymbol{x}}_{it} = \{\widetilde{k}_{it}, \widetilde{m}_{it}\}$ collects input expenditures, with capital entering as the value of fixed assets, $\boldsymbol{p}_{xit} = \{p_{k,it}, p_{m,it}\}$ collects input prices, and $B(\cdot)$, with its own parameters $\boldsymbol{\vartheta}$, absorbs the input-price terms (De Loecker et al., 2016).

- *Productivity.* More productive firms produce more and use more inputs, which biases the elasticities upwards.
- *Output prices.* Firms that charge more sell less for the same revenue and use fewer inputs, which biases the elasticities downwards (Bond et al., 2021).
- *Input prices.* Capital and intermediate inputs are observed only as expenditures, so differences in input prices across firms, from market access, input quality, or bargaining, contaminate the elasticities.

Corrections. A control function addresses the first bias: the demand for intermediate inputs reveals productivity and can be inverted to recover it (Ackerberg et al., 2015; Olley and Pakes, 1996). The EAP addresses the second: it reports prices

and quantities separately, so I deflate revenue by the firm's output price and estimate a quantity production function (De Ridder et al., 2026). For the third, sector and location fixed effects absorb systematic differences in input prices, under the assumption that firms take input prices as given. A second control function, following De Loecker et al. (2016) and Wong (2026), uses the output price as a proxy for input quality: under a broad class of demand systems, firms that use better inputs sell at higher prices. Input prices are then a function of observables, $\boldsymbol{p}_{xit} = \boldsymbol{p}_x(p_{it}, \boldsymbol{Z}_{it})$, where $\boldsymbol{Z}_{it}$ contains sector-by-location fixed effects and controls for markups and markdowns: the firm's revenue share in its five-digit industry and its employment share in its local labour market.

Estimating equation. Substituting the input-price function into (S4.5), and writing $B(\cdot)$ as a flexible basis in $(p_{it}, \boldsymbol{Z}_{it})$ and their interactions with $\{1, \widetilde{\boldsymbol{x}}_{it}, l_{it}\}$ (De Loecker et al., 2016), gives the equation taken to the data:

$$q_{it} = f(\widetilde{k}_{it}, l_{it}, \widetilde{m}_{it}; \beta) + B\left[(p_{it}, \boldsymbol{Z}_{it}) \times \{1, \widetilde{\boldsymbol{x}}_{it}, l_{it}\}; \beta, \boldsymbol{\vartheta}\right] + \varphi_{it} + \varepsilon^q_{it}. \tag{S4.6}$$

I estimate it in two steps, following Ackerberg et al. (2015) and De Ridder et al. (2026). The first step separates measurement error from productivity. The second estimates $(\beta, \boldsymbol{\vartheta})$ together with the law of motion of productivity.

Step 1: measurement error. Productivity is recovered by inverting the demand for intermediate inputs,

$$\varphi_{it} = m^{-1}(\widetilde{k}_{it}, l_{it}, \widetilde{m}_{it}, p_{it}, s^Y_{it}, \boldsymbol{Z}_{it}). \tag{S4.7}$$

The inversion requires that productivity is the only unobserved variable in the demand for intermediate inputs, and that this demand rises with productivity at given controls (Olley and Pakes, 1996). Every argument of m^{-1} must therefore

be observed. The EAP supplies output prices. Markups are not observed, so I control for them with the firm's revenue share in its five-digit industry and year, s^{Y}_{it} (De Ridder et al., 2026), since a firm's relative size determines its markup (Edmond et al., 2023). Year fixed effects absorb common movements in the prices of intermediate inputs. Substituting (S4.7) into (S4.6) gives

$$q_{it} = \mathcal{P}(\widetilde{k}_{it}, l_{it}, \widetilde{m}_{it}, p_{it}, s^{Y}_{it}, \boldsymbol{Z}_{it}) + \varepsilon^{q}_{it}, \tag{S4.8}$$

which I estimate by ordinary least squares. Here $\mathcal{P}$ is a polynomial in log capital, log intermediate-input expenditure, log efficiency hours, the log output price, the revenue share in the five-digit industry, and the employment share in the local labour market, with year and industry-by-location fixed effects entering linearly. The local labour market in this step is the five-digit industry by commuting zone. The polynomial is of second order in industries 20, 27, 29, and 30, and of third order in industries 13, 22, 23, 24, 25, 26, 28, and 32. Industry 29 uses five-digit-industry by region by year fixed effects in the first step, and controls for firm age in both steps.

Step 2: elasticities. With $\hat{q}_{it}$ the fitted output from the first step, productivity at any candidate $(\beta, \boldsymbol{\vartheta})$ is

$$\varphi_{it}(\beta, \boldsymbol{\vartheta}) = \hat{q}_{it} - f(\widetilde{k}_{it}, l_{it}, \widetilde{m}_{it}; \beta) - B\big[(p_{it}, \boldsymbol{Z}_{it}) \times \{1, \widetilde{\boldsymbol{x}}_{it}, l_{it}\}; \beta, \boldsymbol{\vartheta}\big]. \tag{S4.9}$$

The law of motion (S4.4) is linear in every industry, and its innovation is uncorrelated with predetermined variables:

$$\mathbb{E}\big[\varepsilon^{\varphi}_{it}(\beta, \boldsymbol{\vartheta})\, \mathbf{X}_{it}\big] = 0.$$

The translog f contains the first- and second-order terms in $(\widetilde{k}_{it}, l_{it}, \widetilde{m}_{it})$ and their three-way interaction, so β has ten parameters. The basis $B(\cdot)$ is of first order

in every industry: it interacts the output price and the two share controls with $\{1, \widetilde{k}_{it}, \widetilde{m}_{it}, l_{it}\}$, so $\boldsymbol{\vartheta}$ has twelve parameters (sixteen in industry 29, where firm age is a further control). The share controls enter this step in logs. The fixed effects in $\boldsymbol{Z}_{it}$ are removed by a within-transformation and do not enter these interactions. The instruments $\mathbf{X}_{it}$ are current fixed assets; the lagged inputs with their squares, two-way interactions and three-way interaction; and the lagged basis of $B(\cdot)$, with lagged export status and firm age added to the share controls. This gives 31 instruments for 22 parameters (26 in industry 29). The elasticities θ^L_{it} and θ^M_{it} in (S4.2) are the derivatives of f plus those of $B(\cdot)$ with respect to l_{it} and $\widetilde{m}_{it}$, evaluated at the estimates. For each candidate $(\beta, \boldsymbol{\vartheta})$, I compute productivity, recover the innovation from the law of motion, and evaluate the moment conditions; the estimates minimise their quadratic form.

S4.3.3 Results

Levels and dispersion. Table S4.3 reports the markdowns by industry. The median markdown is 0.67 and the mean is 0.77: at the median firm, a worker is paid 67 cents per euro of marginal revenue product. The median markdown is below one in eleven of the twelve industries, and ranges from 0.45 in motor vehicles to 0.88 in other transport equipment; the exception is other manufacturing. Markdowns also differ within industries, with an aggregate interquartile range of 0.38.

Firm size. Figure S4.2 shows that labour market power rises with firm size: log markdowns fall with log revenue and with log employment. Controlling for local labour market by year fixed effects, a 10 per cent larger revenue goes with a 1.1 per cent lower markdown (elasticity −0.11, standard error 0.0016), and a 10 per cent larger workforce with a 0.7 per cent lower markdown (−0.07, standard error 0.0018).[3]

[3] With local labour markets defined as three-digit industry by commuting zone, the elasticities are −0.11 for revenue and −0.07 for employment.

Table S4.3: ESTIMATES OF MARKDOWNS BY INDUSTRY

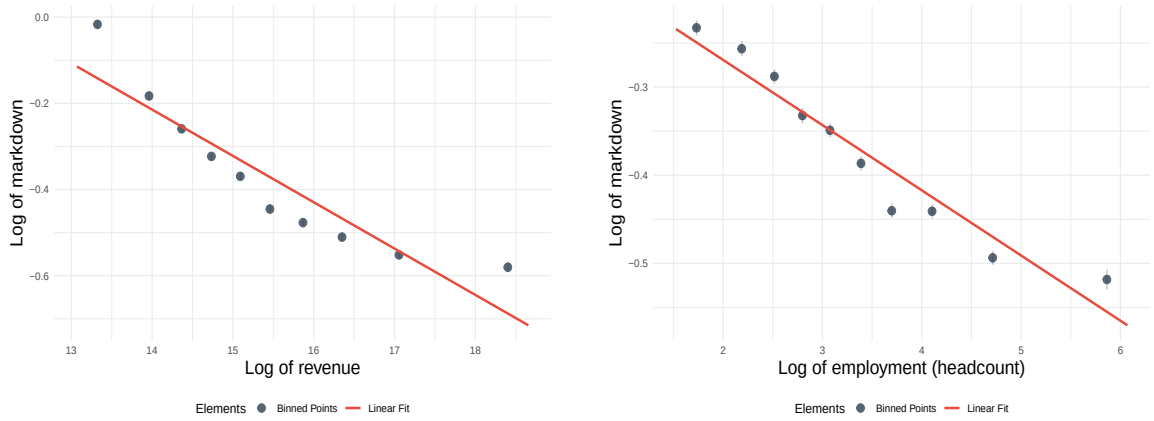
Code	Industry	Mean	Median	IQR
13	Textiles	0.57	0.47	0.28
20	Chemicals	0.59	0.63	0.37
22	Rubber and plastic products	0.54	0.50	0.23
23	Other non-metallic mineral products	0.81	0.73	0.39
24	Basic metals	0.70	0.66	0.37
25	Fabricated metal products	0.83	0.78	0.35
26	Computer and optical products	1.06	0.87	0.61
27	Electrical equipment	0.59	0.56	0.22
28	Machinery	0.72	0.71	0.23
29	Motor vehicles	0.51	0.45	0.25
30	Other transport equipment	1.69	0.88	0.90
32	Other manufacturing	2.19	1.73	1.21
	Aggregate	0.77	0.67	0.38

Notes: I estimate a translog production function separately for each two-digit manufacturing industry listed in the table, using a two-step procedure: (i) isolate measurement error; (ii) recover the key translog parameters. Additional details are provided in Section S4.3.2. Markdowns are winsorised at the 1st and 99th percentiles within each industry; the statistics are over the firm-years of the production-function sample.

Automation. I sort firm-years from 2010 to 2019 into deciles of the production-function markdown, from the least labour market power in bin 1, the highest markdowns, to the most in bin 10, and estimate regression (36) with these deciles. The bin boundaries are computed once on the pooled sample. The sample keeps market-year cells with at least two firms and variation in markdowns within the cell, and standard errors are clustered by firm. For the extensive margin the outcome is the adoption indicator (S4.1). For the intensive margin the sample is restricted to adopters and the outcome is the log of automation intensity, $\log(K^A_{it}/\text{VA}_{i,t-1})$.

Panel (a) of Figure 2 shows that, with the production-function markdown, the probability of holding automation capital rises with labour market power within a

Figure S4.2: SIZE-BASED LABOUR MARKET POWER



(a) Log of Revenue

(b) Log of Employment (headcounts)

Notes: The figures show binscatter plots with data divided into 10 quantile-based bins. Panel (a) has log of revenues on the x-axis and Panel (b) has log of employment on the x-axis. Log of markdowns is on the y-axis. Each bin displays mean values, and linear regression fits are overlaid to illustrate the relationship between markdowns and firm size.

local labour market. In the bottom decile, 40 per cent of firm-years hold a positive stock. Relative to that decile, firms in the fifth decile are 15.8 percentage points more likely to hold automation capital (standard error 1.8), the gap peaks at 36.9 points in the ninth decile (standard error 2.2), and firms in the top decile are 32.0 points more likely to adopt (standard error 2.3).

Panel (b) shows that, among adopters, those with more labour market power also hold more automation capital per euro of value added. Relative to adopters in the bottom decile of the same market and year, intensity is higher by 0.62 log points in the fifth decile (95 per cent confidence interval 0.16 to 1.08) and by 1.27 log points in the top decile (0.74 to 1.81), so the top decile holds about 3.6 times as much automation capital per euro of lagged value added. The differences are significant from the fourth decile onwards.

Both margins therefore show the pattern of Section 5.2, with a measure of labour market power that does not use the model's labour supply curve.

S5 Estimation

Simulation and bootstrap. The standard shocks behind (ϕ_{ih}, T_i, b_i) are drawn once and transformed at every candidate $\boldsymbol{\psi}$, so the objective (39) is a deterministic function of the parameters. Each evaluation uses one set of draws ($S = 1$). To measure the simulation noise this leaves in $\mathbf{m}(\hat{\boldsymbol{\psi}})$, I re-solve the model at $\hat{\boldsymbol{\psi}}$ under 100 independent sets of standard shocks; the covariance of the resulting moments is $\hat{\mathbf{C}}_{\text{sim}}$ in (41). Relative to the data standard error, the simulation standard deviation is about 0.5 for (M1), 0.8 for (M2), 1.3 for (M3), between 0.9 and 1.0 for (M4), and between 1.1 and 1.5 for (M5), similar under the three schedules. The draw also moves the point estimates. To first order, fitting the moments averaged over the 100 draws would lower $\hat{\upsilon}$ and $\hat{c}_k$ by about one standard error of Table 1 and $\hat{\mathcal{R}}_{\phi T}$ by about 0.8 of one standard error, and would leave $\hat{\sigma}_\phi$ and $\hat{\iota}$ essentially unchanged. Each bootstrap replication draws J local labour markets with replacement from the J observed markets, keeps every firm in a drawn market, and recomputes the full moment vector. Replications in which any moment is undefined are discarded and redrawn until 1,000 are obtained.

S5.1 Local identification at the estimates

Let $\mathbf{D}$ be the Jacobian of the five model moments with respect to the five estimated parameters at $\hat{\boldsymbol{\psi}}$, computed by central finite differences with the simulation draws held fixed. Table S5.1 reports two diagnostics under each task schedule.

The first is the matrix $\mathbf{W}^{1/2}\mathbf{D}\,\mathrm{diag}(s)$, in which each moment is measured in units of its standard error and each parameter in units of its scale s_r (the estimate itself, and 0.1 for $\mathcal{R}_{\phi T}$). Its singular values are bounded away from zero and it has full rank, 5, under every schedule, so $\mathbf{D}'\mathbf{W}\mathbf{D}$ is invertible and the estimates are locally identified.

The smallest singular value, between 3.2 and 3.4, says that a move of one scale

Table S5.1: LOCAL IDENTIFICATION

Moment	σ_ϕ	ι	$\mathcal{R}_{\phi T}$	υ	c_k
Decreasing displacement elasticity					
(M1)	39	48	6	2	4
(M2)	57	37	3	2	1
(M3)	51	43	1	2	3
(M4)	32	4	4	59	1
(M5)	9	3	0	26	62
Weakest direction	−0.01	0.02	1.00	−0.06	−0.02
Singular values: 138.6, 99.5, 56.8, 21.9, 3.2					
Constant displacement elasticity					
(M1)	40	49	6	1	3
(M2)	58	36	3	1	1
(M3)	52	44	1	1	2
(M4)	21	8	3	57	11
(M5)	10	2	0	29	59
Weakest direction	−0.01	0.02	1.00	−0.05	−0.02
Singular values: 139.8, 100.1, 56.9, 20.2, 3.3					
Increasing displacement elasticity					
(M1)	41	52	6	1	0
(M2)	62	34	3	1	0
(M3)	54	44	1	1	0
(M4)	12	11	2	59	16
(M5)	3	1	0	41	54
Weakest direction	−0.01	0.02	1.00	−0.06	−0.04
Singular values: 131.1, 104.1, 57.0, 9.2, 3.4					

Notes: Each row gives the share, in per cent, of each parameter in the response of the moment, $100\,|D_{kr}|\,s_r / \sum_{r'} |D_{kr'}|\,s_{r'}$, where D_{kr} is the derivative of moment k with respect to parameter r at the estimate and s_r is the parameter's scale (the estimate, and 0.1 for $\mathcal{R}_{\phi T}$). Rows sum to 100. The Jacobian has rank 5 under each schedule, and the condition number (largest over smallest singular value) is 43, 43 and 39. Singular values are those of $\mathbf{W}^{1/2}\mathbf{D}\,\mathrm{diag}(s)$, so each is a change in the moments, in standard errors, per unit-length move in scaled parameter space; they are listed from largest to smallest. The weakest direction is the right singular vector of the smallest singular value, with one entry per parameter in scale units and unit length. Moment labels as in Table 2.

unit in the worst-determined direction of the parameter space still changes the moments by more than three standard errors. That direction is almost entirely $\mathcal{R}_{\phi T}$ under every schedule: its $\mathcal{R}_{\phi T}$ entry is 1.00 and no other entry exceeds 0.06 in absolute value (row "Weakest direction" in Table S5.1).

The second diagnostic is the share of each parameter in the response of each moment: the entry in row k and column r is $100\,|D_{kr}|\,s_r / \sum_{r'} |D_{kr'}|\,s_{r'}$. The dispersion moments (M1)–(M3) respond mostly to σ_ϕ and ι, the adoption slope (M4) mostly to υ, and the adoption rate (M5) mostly to c_k, under all three schedules.

The shares of $\mathcal{R}_{\phi T}$ are small because its scale is small, not because it is weakly identified: it is the only parameter that moves the two variances in opposite directions, with $\partial(\text{M1})/\partial\mathcal{R}_{\phi T} < 0$, $\partial(\text{M2})/\partial\mathcal{R}_{\phi T} > 0$ and $\partial(\text{M3})/\partial\mathcal{R}_{\phi T}$ small under every schedule.

The sensitivity of the estimate to the data, $\partial\hat{\boldsymbol{\psi}}/\partial\mathbf{m} = (\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}$ (Andrews et al., 2017), confirms it. Measured in standard errors of both the moment and the estimate, a one-standard-error rise in the wage variance (M1) lowers $\hat{\mathcal{R}}_{\phi T}$ by 1.0 standard error, a rise in the employment variance (M2) raises it by 0.5, the covariance (M3) moves it by 0.1, and the two adoption moments move it by less than 0.05, with the same values under all three schedules.

S5.2 Moment construction

This subsection records the measurement conventions behind the moments in Table 2. The moments use the firms of the panel of Section S4.1.3 in 2019. The estimation targets the five moments (M1)–(M5).

Variables. Employment l_i^{data} is the firm's number of workers. It is measured in worker bodies rather than in the efficiency hours used for the production-function estimation of Section S4.3, because the model's l_i is a measure of workers. The employment peg $\mathcal{L}$ of Section 5.4 is the corresponding 2019 total. The wage

bill $\mathcal{B}_i^{\text{data}}$ is deflated. The average wage is $w_i^{\text{data}} = \mathcal{B}_i^{\text{data}}/l_i^{\text{data}}$, the markdown $\delta_i^{\text{data}} = \delta(s_{i|j}^{\text{data}})$ evaluates the model's markdown schedule at the within-market employment share defined in Section 2.1, and the marginal labour cost is $\omega_i^{\text{data}} = w_i^{\text{data}}/\delta_i^{\text{data}}$.

Adoption state. The adoption indicator d_i^{data} equals one when firm i holds a positive stock of imported automation capital in 2019, where the stock (35) accumulates deflated automation imports, depreciating each purchase linearly to zero over eight years (Section S4.1.2). Adoption is not an absorbing state: a firm ceases to count as an adopter once its acquisitions have fully aged out.

Regression moment. Regression (40) is estimated by within-market ordinary least squares on the 2019 firms with a defined adoption state and a finite marginal labour cost, restricted to markets containing at least two such firms and exhibiting within-market variation in $\log \omega_i^{\text{data}}$. The regression is reproduced in each simulated economy.

Sample and weighting. All moments are unweighted across firms, and no observation is trimmed or winsorised beyond the panel construction of Section S4.1.3. Let $\mathcal{I}_2^{2019}$ be the firms in markets containing at least two valid observations, $\mathcal{I}^{2019}$ the 2019 firms of the panel, and u_i and v_i the deviations of $\log w_i$ and $\log l_i$ from their means over the firms of $\mathcal{I}_2^{2019}$ in firm i's market. Then

$$\begin{gathered}(\text{M1}) = \frac{\sum_{i\in\mathcal{I}_2^{2019}} u_i^2}{|\mathcal{I}_2^{2019}| - 1}, \qquad (\text{M2}) = \frac{\sum_{i\in\mathcal{I}_2^{2019}} v_i^2}{|\mathcal{I}_2^{2019}| - 1}, \qquad (\text{M3}) = \frac{\sum_{i\in\mathcal{I}_2^{2019}} u_i v_i}{|\mathcal{I}_2^{2019}| - 1}, \\ (\text{M4}) = \hat{\beta}_4, \qquad (\text{M5}) = \frac{1}{|\mathcal{I}^{2019}|} \sum_{i\in\mathcal{I}^{2019}} d_i,\end{gathered} \tag{S5.1}$$

with $\hat{\beta}_4$ from (40). The simulated counterparts use the identical formulas, so the convention is common to both sides of (39).

Moment (M5) uses all 2019 firms of the panel. Computing the aggregate adoption rate requires neither multiple firms per market nor variation within markets, so firms in single-firm markets remain in its sample. Every firm in the 2019 sample is assigned to a local labour market, so (M5), the simulated market structure and the bootstrap use the same 15,189 firms in 9,326 markets.

S5.3 The cut in the automation price

In each estimated oligopsonistic economy, I lower the automation price to increase aggregate automation capital by 80 per cent, matching the projected growth in the number of industrial robots in France between 2019 and 2029.

Table S5.2 reports the number of operational industrial robots in France from International Federation of Robotics (2025). I project this number to 2029 using its average annual growth rate over 2022–2024:

$$g = \left(\frac{S_{2024}}{S_{2022}}\right)^{1/2} - 1 = 4.63\%, \qquad q = \frac{S_{2024}\,(1+g)^5}{S_{2019}} \approx 1.80,$$

where S_t denotes the number of operational robots at the end of year t.

Let K^0 denote aggregate automation capital in the estimated 2019 economy and $K(\Delta)$ its equilibrium value after reducing c_k by a proportion Δ. Holding all other parameters, firm draws and market structure fixed, I solve for

$$\frac{K(\Delta^*)}{K^0} = q.$$

Each task schedule implies a different price reduction, reported in Table 4. I apply the same proportional price reduction to the corresponding economy without labour markdowns.

Table S5.2: OPERATIONAL INDUSTRIAL ROBOTS IN FRANCE

	2019	2020	2021	2022	2023	2024
Robots at year end	42,054	44,817	49,914	55,244	58,572	60,480
Annual growth (%)		6.6	11.4	10.7	6.0	3.3

Notes: Operational industrial robots at the end of each year, from International Federation of Robotics (2025). The 2019 and 2020 figures are from the October 2023 edition, the later figures from the October 2025 edition.

S6 Sensitivity analysis

This section reports the specifications behind Section 6.3. Each specification is estimated by (39) with the moments, weighting matrix and externally calibrated parameters of Section 5, and both experiments of Section 6 are repeated on the estimated economy. Welfare uses the estimated 2019 economy of each specification as reference.

S6.1 Task schedule

The baseline schedules share $a = 0.5$. Here a is set to 0.1 and to 0.9, and under each value β_R is chosen so that the displacement elasticity $h(\tilde{x}) = [a + \beta_R(1 - \tilde{x})]^{-1}$ decreases, stays constant or increases, as in the baseline. The values are $\beta_R = -a/2$, 0 and $2a$, which keep the ratio $h(0)/h(1) = a/(a + \beta_R)$ at its baseline values of 2, 1 and $1/3$, so that a moves only the level of the elasticity, $h(1) = 1/a$. Table S6.1 reports the estimates, Table S6.2 the fit, and Table S6.3 the two experiments.

Table S6.1: ESTIMATES UNDER ALTERNATIVE TASK SCHEDULES

	$a = 0.1$			$a = 0.9$		
	$h' < 0$	$h' = 0$	$h' > 0$	$h' < 0$	$h' = 0$	$h' > 0$
Fixed schedule parameters						
a	0.1	0.1	0.1	0.9	0.9	0.9
β_R	−0.05	0	0.2	−0.45	0	1.8
Estimated						
σ_ϕ	0.637 (0.013)	0.627 (0.013)	0.594 (0.013)	0.553 (0.009)	0.546 (0.009)	0.534 (0.009)
ι	0.759 (0.020)	0.758 (0.020)	0.757 (0.020)	0.754 (0.020)	0.754 (0.020)	0.753 (0.020)
$\mathcal{R}_{\phi T}$	0.247 (0.037)	0.258 (0.037)	0.299 (0.034)	0.393 (0.027)	0.402 (0.027)	0.416 (0.027)
υ	0.351 (0.011)	0.362 (0.011)	0.416 (0.018)	0.591 (0.046)	0.659 (0.059)	0.817 (0.081)
c_k	1.349 (0.024)	1.370 (0.024)	1.492 (0.031)	0.657 (0.030)	0.811 (0.048)	1.811 (0.164)

Notes: Each column is estimated by (39) on the five moments of Table 2, with the externally calibrated parameters and normalisations of Table 1. Standard errors from (41) in parentheses.

Table S6.2: MODEL FIT UNDER ALTERNATIVE TASK SCHEDULES

		$a = 0.1$			$a = 0.9$		
	Data	$h' < 0$	$h' = 0$	$h' > 0$	$h' < 0$	$h' = 0$	$h' > 0$
(M1) Variance of log wages	0.048	0.048	0.048	0.048	0.048	0.048	0.048
(M2) Variance of log employment	0.765	0.765	0.765	0.765	0.765	0.765	0.765
(M3) Covariance	0.020	0.020	0.020	0.020	0.020	0.020	0.020
(M4) Adoption slope	0.438	0.438	0.438	0.438	0.438	0.438	0.438
(M5) Adoption rate	0.437	0.437	0.437	0.437	0.437	0.437	0.437

Notes: Moments as in Table 2; (M1)–(M3) are within-market moments and (M4) is the within-market slope of adoption on the log marginal labour cost. The Data column repeats Table 2. The other columns report the simulated moments at the estimates of Table S6.1.

Table S6.3: COUNTERFACTUALS UNDER ALTERNATIVE TASK SCHEDULES

	$a = 0.1$			$a = 0.9$		
	$h' < 0$	$h' = 0$	$h' > 0$	$h' < 0$	$h' = 0$	$h' > 0$
Panel A. Welfare cost of labour market power (% of 2019 C)						
Endogenous automation, G	3.69	3.99	5.23	7.70	8.52	10.42
No automation, G^0	1.91	2.18	3.43	6.61	7.59	10.05
Difference (% of G)	48.3	45.3	34.4	14.2	10.9	3.6
Panel B. Cheaper automation (welfare cost in % of 2019 C)						
Automation-price reduction (%)	11.3	11.3	11.6	13.9	14.7	18.0
Welfare cost, 2019 price	3.69	3.99	5.23	7.70	8.52	10.42
Welfare cost, lower price	3.86	4.18	5.50	7.92	8.74	10.52
Increase (% of cost at 2019 price)	4.6	4.8	5.2	2.8	2.6	1.0
Gain from cheaper automation in the oligopsonistic economy (% of 2019 C)						
Social welfare $\mathcal{S}$	40.81	39.37	33.42	22.17	18.13	8.30
Workers' welfare $\widetilde{\mathcal{W}}$	−0.04	−0.02	0.08	0.58	0.63	0.54
Owners' welfare $\mathcal{W}^f$	40.85	39.40	33.34	21.58	17.50	7.77

Notes: Panel A repeats the experiment of Table 3: G and G^0 are the welfare gains from removing labour markdowns with endogenous automation and with every automated share held at zero, and the difference is $(G - G^0)/G$. Panel B repeats the experiment of Table 4, with the price cut set for each column to deliver the same growth in automation capital as in the baseline. Workers' and owners' gains sum to the social welfare gain up to rounding.

S6.2 Market definition

The baseline defines a local labour market as a five-digit industry within a commuting zone. Here a market is a three-digit industry within a commuting zone. The market structure J, $\{N_j\}$ and the within-market shares change with the definition, and so do the markdowns $\delta(s_{i|j})$, the marginal labour costs and the within-market moments (M1)–(M4), which are recomputed on the 2019 cross-section; the adoption rate (M5) does not depend on the definition. The task schedules are the three baseline schedules, with $a = 0.5$. Table S6.4 reports the estimates and fit, and Table S6.5 the two experiments.

Table S6.4: ESTIMATES AND FIT WITH THREE-DIGIT MARKETS

		Displacement elasticity		
		Decreasing	Constant	Increasing
Panel A. Estimates				
σ_ϕ		0.567 (0.008)	0.559 (0.008)	0.540 (0.007)
ι		0.741 (0.017)	0.741 (0.017)	0.741 (0.017)
$\mathcal{R}_{\phi T}$		0.324 (0.027)	0.338 (0.027)	0.369 (0.026)
υ		0.486 (0.024)	0.535 (0.025)	0.692 (0.050)
c_k		0.972 (0.024)	1.089 (0.029)	1.755 (0.094)
Panel B. Moments				
	Data	Model		
(M1) Variance of log wages	0.055 (0.003)	0.055	0.055	0.055
(M2) Variance of log employment	0.880 (0.018)	0.880	0.880	0.880
(M3) Covariance	0.023 (0.003)	0.023	0.023	0.023
(M4) Adoption slope	0.437 (0.021)	0.437	0.437	0.437
(M5) Adoption rate	0.437 (0.006)	0.437	0.437	0.437

Notes: A local labour market is a three-digit industry within a commuting zone. The task schedules have $a = 0.5$ and $\beta_R = -0.25$, 0 and 1. Moments as in Table 2, recomputed with the three-digit definition; data standard errors, in parentheses, from the cluster bootstrap of Section 5.5, clustered by three-digit market. Parameter standard errors from (41) in parentheses.

Table S6.5: COUNTERFACTUALS WITH THREE-DIGIT MARKETS

	Displacement elasticity		
	Decreasing	Constant	Increasing
Panel A. Welfare cost of labour market power (% of 2019 C)			
Endogenous automation, G	7.36	8.06	9.94
No automation, G^0	5.90	6.73	9.11
Difference (% of G)	19.8	16.5	8.4
Panel B. Cheaper automation (welfare cost in % of 2019 C)			
Automation-price reduction (%)	12.6	13.1	15.0
Welfare cost, 2019 price	7.36	8.06	9.94
Welfare cost, lower price	7.66	8.38	10.22
Increase (% of cost at 2019 price)	4.0	3.9	2.8
Gain from cheaper automation in the oligopsonistic economy (% of 2019 C)			
Social welfare $\mathcal{S}$	25.57	22.40	13.54
Workers' welfare $\widetilde{\mathcal{W}}$	0.43	0.52	0.65
Owners' welfare $\mathcal{W}^f$	25.14	21.88	12.89

Notes: As Table S6.3, for the economies of Table S6.4.

References

[1] **Acemoglu, D., and P. Restrepo.** 2022. "Demographics and Automation." *The Review of Economic Studies* 89 (1): 1–44.

[2] **Ackerberg, D. A., K. Caves, and G. Frazer.** 2015. "Identification Properties of Recent Production Function Estimators." *Econometrica* 83 (6): 2411–2451.

[3] **Andrews, I., M. Gentzkow, and J. M. Shapiro.** 2017. "Measuring the Sensitivity of Parameter Estimates to Estimation Moments." *The Quarterly Journal of Economics* 132 (4): 1553–1592.

[4] **Azkarate-Askasua, M., and M. Zerecero.** 2025. "Union and Firm Labor Market Power." Unpublished manuscript, SSRN Working Paper 4323492, June 2025 version, Website.

[5] **Bergounhon, F., C. Lenoir, and I. Mejean.** 2018. "A Guideline to French Firm-Level Trade Data." Unpublished manuscript, CREST, September 2018 version,

Website.

[6] **Bond, S., A. Hashemi, G. Kaplan, and P. Zoch.** 2021. "Some Unpleasant Markup Arithmetic: Production Function Elasticities and Their Estimation From Production Data." *Journal of Monetary Economics* 121: 1–14.

[7] **Chan, M., S. Salgado, and M. Xu.** 2026. "Heterogeneous Passthrough From TFP to Wages." *Journal of Labor Economics*, Forthcoming.

[8] **De Loecker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik.** 2016. "Prices, Markups, and Trade Reform." *Econometrica* 84 (2): 445–510.

[9] **De Ridder, M., B. Grassi, and G. Morzenti.** 2026. "The Hitchhiker's Guide to Markup Estimation: Assessing Estimates From Financial Data." *Econometrica* 94 (1): 137–168.

[10] **Edmond, C., V. Midrigan, and D. Y. Xu.** 2023. "How Costly Are Markups?" *Journal of Political Economy* 131 (7): 1619–1675.

[11] **International Federation of Robotics.** 2025. "World Robotics: Industrial Robots." Operational stock of industrial robots at year-end by country, as tabulated by the Japan Robot Association (editions of October 2023 and October 2025).

[12] **Olley, G. S., and A. Pakes.** 1996. "The Dynamics of Productivity in the Telecommunications Equipment Industry." *Econometrica* 64 (6): 1263–1297.

[13] **Wong, H. C.** 2026. "Understanding High-Wage Firms: Monopoly, Monopsony, and Bargaining Power." *American Economic Review* 116 (7): 2504–2541.

[14] **Yeh, C., C. Macaluso, and B. Hershbein.** 2022. "Monopsony in the US Labor Market." *American Economic Review* 112 (7): 2099–2138.